Lecture on Optimization-based High Order Sliding Mode Control and Applications.
Part 4: Automotive Applications

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Summary

• Traction Control with Fastest Acceleration/Deceleration via Second Order Sliding Mode Control

• Vehicles Platooning via Second Order Sliding Mode Control

• Braking Control of Two-Wheeled Vehicles via Switched Second Order Sliding Mode Control

• Torque Vectoring Control in Fully Electric Vehicles via SMC
Traction Control with Fastest Acceleration/Deceleration Via Second Order Sliding Mode Control
Traction Force Control

• The traction force control is an automatic driver assistance system which increases vehicle drivability especially in difficult wheater conditions, allowing anti-skid braking and anti-spin acceleration.

• The traction force produced by a vehicle is strongly influenced by road conditions.

• It is necessary to design a robust traction force controller taking into account the time-varying tire/road interaction.

• As a further requirement, the designed control law has to prevent the generation of vibrations which could increase the discomfort index.
The Vehicle Model

\[ m\dot{v}_x = \left[ F_{xf} (\lambda_f) + F_{xr} (\lambda_r) \right] - F_{loss} \]
\[ J_i \dot{\omega}_i = T_i - r_i F_{xi} (\lambda_i) \]
\[ \dot{\lambda}_i = f_i + h_i (v_x) T_i \quad i \in \{ f, r \} \]
\[ F_{zf} = \frac{l_r mg - l_h m\dot{v}_x}{2(l_f + l_h)} \]
\[ F_{zr} = \frac{l_f mg + l_h m\dot{v}_x}{2(l_f + l_h)} \]

\[ F_{loss} = F_{air} (v_x) + F_{roll} \]
\[ f_i = -\frac{\dot{v}_x}{\omega_i r_i} - \frac{v_x}{J_i \omega_i^2} F_{xi} \]
\[ h_i (v_x) = \frac{v_x r_i}{J_i (\omega_i r_i)^2} \]

- Bicycle model ("single-track model")
- Roll and yaw dynamics, lateral and vertical motions, and actuators dynamics are neglected
- To design the controller, the normal forces are assumed to be constant (this assumption is removed in simulations)
The Traction Force Control Problem

- The traction force $F_x$ depends on the tire/road interaction, (which depends on the road conditions), on the normal force $F_z$, and on the “slip ratio” $\lambda$

$$\lambda := \frac{\omega r - v_x}{\max(\omega r, v_x)}$$

- The “traction controller” can be realized as a “slip controller”: to control $\lambda$ so that the desired traction force is produced, by using the torques acting on the front and rear wheel, $T_f$, $T_r$, as control variables.

- To design the slip controller it is necessary to model the tire/road interaction.
# Tire/Road Interaction Models

<table>
<thead>
<tr>
<th>Year</th>
<th>Model Name</th>
<th>Model Properties</th>
<th>Features</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>2. Has lots of revised formula</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>3. Can employ different factors</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>2. Has some revised formula</td>
</tr>
<tr>
<td>1994</td>
<td>Rill Model</td>
<td>Semi-Empirical</td>
<td>Easy to identify</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>2. Can produce smooth transition around zero velocity</td>
</tr>
<tr>
<td>1991</td>
<td>Bliman-Sorine Model</td>
<td>Analytical</td>
<td>Can capture the Strubeck effect in addition to Dahl model</td>
</tr>
<tr>
<td>1995</td>
<td>LuGre Model</td>
<td>Analytical</td>
<td>Can combine pre-sliding &amp; sliding in addition to Bliman-Sorine Model</td>
</tr>
</tbody>
</table>


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The well-known "Magic Formula" by Bakker-Pacejka models the traction force as a function of the slip ratio parametrized by the tire-road adhesion coefficient $\mu_p$.

$$F_x = \mu_p \cdot f_t(\lambda, F_z)$$

$\mu_p$ depends on road conditions and needs to be estimated.
The control objective for the FADC problem is to maximize the generated traction force.

The desired slip ratio $\lambda_d$ is the abscissa of the extremal value of the $\lambda - F_x$ curve corresponding to the actual road condition.

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Solution of the FADC Problem

To solve the **FADC problem** the control scheme must accomplish the following tasks:

1. **To estimate on-line the adhesion coefficient** $\mu_p$ to identify the actual $\lambda - F_x$ curve.

2. **To calculate the desired slip ratio** $\lambda_d$ as the abscissa of the extremal value of the actual $\lambda - F_x$ curve.

3. **To design a control law** that makes the actual slip ratio $\lambda$ track the desired value $\lambda_d$.
The Adhesion Coefficient Estimate 1/2

The adhesion coefficient can be estimated using a recursive least square (RLS) technique with “forgetting factor”.

The objective is to find the value $\hat{\mu}_p$ that minimizes the following cost

$$J = \int_0^t e^{-\int_0^t \rho(r)dr} \left\| y(\tau) - \hat{\mu}_p(t) \phi(\tau) \right\|^2 d\tau$$

$\rho(t) \geq 0$ forgetting factor

$y(t) = \frac{1}{2} (m \dot{v}_x(t) - F_{loss}(t)) = \mu_p(t) \phi(t)$

$\phi(t) = f_{tf}(t) + f_{tr}(t)$

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The parameter update law is given by

\[ \dot{\hat{\mu}}_p = -P(t)\phi(t)e(t) \]

where \( e(t) = \hat{\mu}_p \phi(t) - y(t) \) and \( P(t) \) is the update gain of parameter \( \hat{\mu}_p \).

The update gain can for instance be adjusted as

\[ \dot{P}(t) = \rho(t)P(t) - \phi(t)^2 P(t)^2 \]
The Slip Controller 1/2

The selected **sliding variables** are the “**slip tracking error**” at the front and rear wheel, respectively

\[ s_i = \lambda_i - \lambda_{i_d} = \sigma_{i1} \quad i \in \{f, r\} \]

\[
\begin{cases}
\dot{s}_i = \dot{\sigma}_{i1} = \sigma_{i2} \\
\ddot{s}_i = \ddot{\sigma}_{i2} = f_i + h_i \dot{T}_i - \dot{\lambda}_{i_d} + h_i \dot{T}_i
\end{cases}
\]

**Auxiliary system**

only the knowledge of the **upperbounds** is necessary to design the control law

\[ |\varphi_i(t)| < \Phi_i \quad 0 < \Gamma_{i1} \leq \gamma_i(t) \leq \Gamma_{i2} \]
The auxiliary systems are in a form suitable to apply a second order sliding mode technique so that, the sliding variables and their first time derivatives, 
\[ \sigma_{i1} = s_i, \quad \sigma_{i2} = \dot{s}_i, \]
are steered to zero in a finite time.

As a consequence, the slip error relevant to the front and rear wheel vanishes in a finite time

\[ s_i = \lambda_i - \lambda_{id} = 0 \]
The Sub-Optimal Controller

\[
\dot{T}_i(t) = -\alpha_i V_{iM} \text{sgn}\left(s_i(t) - \frac{1}{2}s_{iM}\right), \quad i \in \{f, r\}
\]

\[
V_{iM} > \max \left(\frac{\Phi_i}{\alpha_i \Gamma_i}, \frac{4\Phi_i}{3\Gamma_i - \alpha_i \Gamma_2}\right) \quad \alpha^*_i \in (0,1] \cap \left(0, \frac{3\Gamma_i}{\Gamma_2}\right)
\]

\[
\alpha_i = \begin{cases} 
\alpha^*_i & \left[x_i(t) - 0.5x_{iM}\right] \cdot \left|x_{iM} - x_i(t)\right| > 0 \\
1 & \text{otherwise}
\end{cases}
\]
The Super Twisting Controller

Control law

\[ T_i(t) = -\eta_i s_i |\rho_i| \text{sgn}(s_i) + u_{i1}(t), \quad i \in \{f, r\} \]

\[ \dot{u}_{i1}(t) = -W_i \text{sgn}(s_i) \]

0 < \rho_i \leq 0.5

\[ W_i > \frac{\Phi_i}{\Gamma_{i1}} \]

\[ \eta_i^2 > \frac{4\Phi_i \Gamma_{i2}(W_i + \Phi_i)}{\Gamma_{i1}^3(W_i - \Phi_i)} \]
Simulation Test

In the simulated scenario, the vehicle performs an acceleration manoeuvre for 6 s starting from the velocity of 20 m/s. The road condition changes at 3 s from dry asphalt to wet asphalt.

<table>
<thead>
<tr>
<th>g</th>
<th>9.81 m/s²</th>
</tr>
</thead>
<tbody>
<tr>
<td>m</td>
<td>1366 kg</td>
</tr>
<tr>
<td>J_{wf}</td>
<td>1.07 kgm²</td>
</tr>
<tr>
<td>J_{wr}</td>
<td>1.07 kgm²</td>
</tr>
<tr>
<td>c_x</td>
<td>0.4</td>
</tr>
<tr>
<td>f_{roll}</td>
<td>0.013</td>
</tr>
<tr>
<td>r_{wf}</td>
<td>0.32 m</td>
</tr>
<tr>
<td>r_{wr}</td>
<td>0.32 m</td>
</tr>
<tr>
<td>v_x(0)</td>
<td>30 m/s</td>
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<tr>
<td>\lambda_f(0)</td>
<td>0.15</td>
</tr>
<tr>
<td>\lambda_r(0)</td>
<td>0.15</td>
</tr>
<tr>
<td>V_{fM} = V_{rM}</td>
<td>2000 N/ms</td>
</tr>
<tr>
<td>\alpha_f^* = \alpha_r^*</td>
<td>1</td>
</tr>
<tr>
<td>\eta_f = \eta_r</td>
<td>100 N/ms</td>
</tr>
<tr>
<td>W_f = W_r</td>
<td>3000 N/ms²</td>
</tr>
<tr>
<td>\rho_f = \rho_r</td>
<td>0.5</td>
</tr>
</tbody>
</table>

The total available braking torque and the engine torque exerted on the driving shaft can be determined from

\[ T_j, \ j \in \{f, r\} \]

For instance, for a front-wheel-driven (FWD) vehicle:

\[
T_f = 0.5T_{\text{shaft}} - 0.3T_{\text{brake}}
\]

\[
T_r = -0.2T_{\text{brake}}
\]
Sub-Optimal: Simulation Results 1/2

Slip at the front wheel

Slip error at the front wheel

Slip at the rear wheel

Slip error at the rear wheel
Sub-Optimal: Simulation Results 2/2

Engine and Braking Torque

Real road condition

Estimated road condition

Vehicle Velocity

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Super Twisting: Simulation Results 1/2

Slip at the front wheel

Slip at the rear wheel

Slip error at the front wheel

Slip error at the rear wheel

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Super Twisting: Simulation Results 2/2

Engine and Braking Torque

Real road condition

Estimated road condition

Vehicle Velocity

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Simulation Results: A Comparison

Braking torque and engine torque with the considered second order sliding mode controllers

Braking torque and engine torque with a conventional first order sliding mode controller

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Vehicles Platooning via Second Order Sliding Mode Control
Control of a Platoon of Vehicles

• The longitudinal control of platoons of vehicles is effective to improve traffic capacity and road safety by reducing traffic dishomogeneities.

• As for traction control, the performances of a “platooning control system” are strongly influenced by road conditions.

• The “platooning control problem” has to be solved in a robust way.

• As a further requirement, also in this case, the designed control law has to prevent the generation of vibrations induced by the controller.
The Platooning Control Problem 1/2

- **Control objective:** make the distance between two subsequent vehicles of the platoon be equal to a pre-specified "safety distance".

\[
d_i = x_{i-1} - x_i = x_{\text{spacing}}(v_{x_i})
\]

- Solving the **platooning problem** even in presence of possible changes of the road conditions requires to re-formulate the problem as a **traction control problem** and take again into account the **tire/road interaction**.
A possible way to determine the “safety distance” is to rely on the following “variable spacing policy”

$$x_{spacing}(v_{x_i}) = S_{d_0} + hv_{x_i}$$

To solve the platooning control problem as a traction control problem means to design a controller capable of generating the traction force $F_x$ suitable to steer to zero the “spacing error”

$$e_i = x_{spacing}(v_{x_i}) - d_i$$
The Traction Control Problem

- The traction force $F_x$ depends on the \textit{tire/road interaction}, (which depends on the road conditions), on the normal force $F_z$, and on the \textit{“slip ratio”} $\lambda$

$$\lambda := \frac{\omega r - v_x}{\max (\omega r, v_x)}$$

- The \textit{“traction control”} can be realized as a \textit{“slip control”}: to control $\lambda$ so that the desired traction force is produced, by using the torques acting on the front and rear wheel, $T_f, T_r$.

- To design the slip control it is necessary to model the \textit{tire/road interaction}. Also in this case the Bakker-Pacejka model can be used.
The Control Scheme

**Outer loop:** on the basis of the **spacing error** the controller determines the **desired traction force**. Relying on this latter and on the current tire/road adhesion coefficient the **desired slip ratios** are calculated. These are references for the inner loop.

**Inner loop:** has the objective of attaining the desired slip ratios by acting on the **torques at the front and rear wheels**.
Sliding variable: the "spacing error" associated with the \( i \)-th vehicle

\[
S_i = e_i = S_{d_0} + h v_{x_i} - x_{i-1} + x_i
\]

To design a "second order sliding mode control law", determine the first and second time derivatives

\[
\dot{S}_i = \dot{e}_i = v_{x_i} - v_{x_{i-1}} + h \dot{v}_{x_i}
\]

\[
\ddot{S}_i = \ddot{e}_i = \ddot{v}_{x_i} - \ddot{v}_{x_{i-1}} + h \ddot{v}_{x_i}
\]
By introducing the **auxiliary variables**

\[
y_{1i} = S_i \quad y_{2i} = \dot{S}_i
\]

the previous differential equations can be rewritten as

\[
\begin{cases}
\dot{y}_{1i} = y_{2i} \\
\dot{y}_{2i} = \epsilon_i + W_i
\end{cases}
\]

where \( |\epsilon_i| = |\dot{v}_{xi} - \dot{v}_{xi-1}| \leq \Gamma_i \) and \( w_i = h\dot{v}_{xi} \)

only the knowledge of the **upperbound** is necessary to design the control law.
It can be proved that, if the **auxiliary control signal** is chosen (for instance) as

\[
    w_i(t) = -W_{M_i} \text{sign}\left\{y_{1i} - \frac{1}{2} y_{1i,\text{MAX}}\right\}, \quad y_{1i,\text{MAX}} = y_{1i} \bigg|_{y_{2i} = 0}
\]

where \(W_{M_i} > 2\Gamma_i\)

then, the sliding variable and its first time derivative,

\(y_{1i} = S_i, \quad y_{2i} = \dot{S}_i\),

are steered to zero in a finite time.

As a consequence, the spacing error between two subsequent vehicles vanishes in a finite time

\[
    S_i = e_i = S_{d_0} + h_n x_i - x_{i-1} + x_i = 0
\]
On the basis of the auxiliary signal $w_i(t)$, the desired traction force (the actual control signal) can be determined as

$$F_{xf_{ides}} + F_{xr_{ides}} = \frac{1}{2} \left( m_i \dot{v}_{x_{ides}} + F_{loss_i} \right)$$

where

$$\dot{v}_{x_{ides}} = \frac{1}{h} \int_{t_0}^{t} w_i(\tau) \, d\tau$$

Note that, as usual in this context, it is assumed that the force distribution is described by

$$F_{xf_{ides}} = l_{r_i} + l_{h_i} \left( \frac{F_{loss_i}}{m_i g} + \mu_i \right)$$

$$F_{xr_{ides}} = l_{f_i} - l_{h_i} \left( \frac{F_{loss_i}}{m_i g} + \mu_i \right)$$
The slip reference for the inner loop of the $i$-th vehicle control system

\[ \hat{\mu}_p \]

\[ F_{xj_i} \]

\[ j \in \{ f, r \} \]

\[ \lambda_{j_{ides}} \]

traction force determined by the longitudinal controller as the force necessary to steer the spacing error between vehicles to zero

slip ratio reference for the inner loop of the $i$-th vehicle control system
The Slip Controller 1/2

Sliding variable: the “slip error”
for each wheel $j$ of the $i$-th vehicle

$$s_j = \lambda_j - \lambda_{j,\text{des}} = \sigma_{j,1}$$

Auxiliary system

$$j \in \{f, r\}$$ (the subscript $i$ is omitted!)

$$\begin{align*}
\dot{s}_j &= \dot{\sigma}_{j,1} = \sigma_{j,2} \\
\ddot{s}_j &= \ddot{\sigma}_{j,2} = f_j + \dot{h}_j(v_x)T_j - \ddot{\lambda}_d + h_j(v_x)\dddot{T}_j
\end{align*}$$

Auxiliary control

$$|F_j(t)| < F_j \quad 0 < G_{j,1} \leq g_j(t) \leq G_{j,2}$$

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The Slip Controller 2/2

It can be proved that, if the **auxiliary control signal** is chosen as

\[
\nu_j(t) = \dot{T}_j(t) = -\alpha_j N_j \text{sgn}\left(\sigma_{1j}(t) - \frac{1}{2} \sigma_{1j,MAX}\right), \quad \sigma_{1j,MAX} = \sigma_{1j}\big|_{\sigma_{2j}=0}
\]

with \[ N_j > \max\left(\frac{F_j}{\alpha_j^* G_{j1}}; \frac{4F_j}{3G_{j1} - \alpha_j^* G_{j2}}\right) \]

and \( j \in \{f, r\} \)

then, the sliding variable and its first time derivative, \( \sigma_{1j} = s_j, \quad \sigma_{2j} = \dot{s}_j \), are steered to zero in a finite time.

As a consequence, the **slip error** relevant to each wheel of the **i-th vehicle** vanishes in a finite time

\[
s_j = \lambda_j - \lambda_{j,\text{des}} = 0
\]
Simulation Test

Two vehicles, a leader and a follower.

**TABLE 1**

**SIMULATION PARAMETERS**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$g$</td>
<td>9.81 m/s$^2$</td>
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</tr>
<tr>
<td>$\lambda_r(0)$</td>
<td>0.01</td>
</tr>
</tbody>
</table>
Simulation Results

Spacing error

Inter-vehicular distance

Slip error at the front wheel

Slip error at the rear wheel

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Simulation Results

The total available braking torque and the engine torque exerted on the driving shaft can be determined from $T_j$, $j \in \{f, r\}$ For instance, for front-wheel-driven (FWD) vehicles:

$$T_f = 0.5T_{\text{shaft}} - 0.3T_{\text{brake}}$$

$$T_r = -0.2T_{\text{brake}}$$
Simulation Results

Braking torque and engine torque with the proposed second order sliding mode controller

Braking torque and engine torque with a conventional first order sliding mode controller

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Simulation Results

Inter-vehicular distance with the designed second order sliding mode controller (ideal sliding mode!)

Inter-vehicular distance with a conventional first order sliding mode controller with continuous approximation of the sign function (quasi sliding mode)
Braking Control of Two-Wheeled Vehicles via Switched Second Order Sliding Mode Control
Braking Control

• Braking control is based on the regulation of the wheel slip (relative speed between wheel and center of mass). In motorbikes the slip dynamics is dependent on the vehicle speed.

• As an alternative to conventional adaptive control, we have proposed to apply the switched formulation of second order sliding mode controllers (S-SOSM), since it can be useful in all contexts where different uncertainties and/or performance specs are associated with different regions of the state space.

MAIN IDEA:
- partition the state-space into different regions
- tune a dedicated S-SOSM controller for each of them
Switched SOSM: state-space partitioning

\[ \mathcal{R}_i : = \left\{ (z_1, z_2) : |z_1| \leq \bar{z}_{1i} \text{ and } |z_2| \leq \bar{z}_{2i} \right\} \]

The state space \( \mathcal{Z} \) is partitioned in \textbf{k regions} \( \mathcal{R}_i, i=1, \ldots, k \), all containing the \textbf{origin}, such that \( \bigcup_i \mathcal{R}_i = \mathcal{Z} \) and with \( \mathcal{R}_{i+1} \subset \mathcal{R}_i \). Let us define as \textbf{switching surfaces} \( \mathcal{S}_i = \partial \mathcal{R}_{i+1}, i=1, \ldots, k-1 \).
Switched SOSM: uncertainty description

- **Case 1: Outermost Region** $Z_1$
  \[ 0 < G_{11} \leq g(z(t)) \leq G_{21} \]
  \[ |f(z(t))| < \mathcal{F}_1(z) \]

- **Case 2: Inner Regions** $Z_i, i=2,\ldots,k$
  \[ 0 < G_{1i}(z) \leq g(z(t)) \leq G_{2i}(z) \]
  \[ |f(z(t))| < \mathcal{F}_i(z) \]
  \[
  G_{1i}(z) = G_{1i}(z) + \overline{G}_{1i} \\
  G_{2i}(z) = G_{2i}(z) + \overline{G}_{2i} \\
  \mathcal{F}_i(z) = \mathcal{F}_i(z)
  \]

- As in the inner regions the state norm can be bounded, one can write:
  \[ 0 < \overline{G}_{1i} \leq g(z(t)) \leq \overline{G}_{2i} \]
  \[ |f(z(t))| < \overline{\mathcal{F}_i} \]
Switched SOSM: control algoritms

- Two different algorithms:
  - **Full switched S-SOSM (FS-SOSM)** → both the gain $V$ and $\alpha^*$ are adapted in the different regions (different uncertainties and different performance specifications)
  - **Gain switched S-SOSM (GS-SOSM)** → only the gain $V$ is varied (uniform uncertainties in the whole state space but different performance specifications)
**Switched SOSM: FS S-SOSM Control Algorithm**

- **Outermost Region** $Z_1$

1. Initialization: for $0 \leq t \leq t_{M1}$

$$v(t) = -\frac{1}{G_{11}} [F_1(z(t)) + \nu] \text{sign}(z_1(0)), \quad t = 0$$

$$v(t) = -\frac{1}{G_{11}} [F_1(z(t)) + \nu] \text{sign}[z_1(t) - z_1(0)], \quad 0 \leq t \leq t_{M1}$$

2. For $t_{Mj} \leq t \leq t_{Mj+1}$ such that $z(t) \in Z_1$

$$v(t) = -V_{Mj} \text{sign}\left(z_1(t) - \beta_j z_{1Mj}\right)$$

$$V_{Mj} = \frac{\pi}{G_{11}} \left[F_1 + \frac{1}{3} \eta^2\right], \quad \pi > 1, \quad \beta_j = \max\left\{\frac{1}{2}, 1 - \frac{\eta^2}{2 [F_1 + G_{21}V_{Mj}]}\right\}$$
Switched SOSM: FS S-SOSM Control (2)

• Inner Regions:

If \( z(t) \in \mathcal{Z}_i \setminus \{p_{1,i}, p_{2,i} \cup p_{5,i}, p_{6,i}\} \), \( i = 2, \ldots, k \)

\[
v(t) = -\alpha_i V_i \text{sign} \left( z_1 - \frac{1}{2} \frac{z_{Max}}{z_{Max}} \right) \quad \alpha_i = \begin{cases} \alpha^*_i & \text{if } \left[ z_1 - \frac{1}{2} \frac{z_{Max}}{z_{Max}} \right] \left[ z_{Max} - z_1 \right] > 0 \\ 1 & \text{else}, \end{cases}
\]

If \( z(t) \in \{p_{1,i}, p_{2,i} \cup p_{5,i}, p_{6,i}\} \), \( i = 2, \ldots, k \)

\[
v_i(t) = -\alpha_i V_i \text{sign} \left( z_1(t) - \frac{1}{2} \frac{z_{Max}}{z_{Max}} \right) \\
v_{i-1}(t) = \alpha_i V_i \text{sign} \left( z_1(t) - \frac{1}{2} \frac{z_{Max}}{z_{Max}} \right),
\]
Braking control of two wheeled vehicles

Main components:

- **Servo-control loop for the actuator**: brake pressure control system

- **Wheel slip estimation (from indirect measurements)**: needed if front and rear brake is used. If front brake only (common on motorcycles), the rear wheel speed gives an estimate of the vehicle speed

- **Slip control**: the goal is to track a wheel slip target

We have considered Braking Control on a straight line (target slip definition “easier”)

Antonella Ferrara

Spring School on SMC, Aussois, June 2015
Braking control of two wheeled vehicles: vehicle model

- The wheel slip in braking is defined as

\[ \lambda_i = \frac{v - \omega_i r}{\omega_i r} \]

\[
\begin{align*}
J \dot{\omega}_f &= r_f F_{xf} - T_{bf} \\
J \dot{\omega}_r &= r_r F_{xr} - T_{br} \\
m \dot{v} &= -F_{xf} - F_{xr}
\end{align*}
\]

\[ F_{x_i} = F_{z_i} \mu(\lambda_i, \alpha_i t, \vartheta), \quad i = \{ f, r \} \]

- Load transfer between front and rear wheels $\rightarrow$ proportional to longitudinal acceleration

\[ F_{zf} = \frac{mg l_r}{l} - \frac{mh}{l} \dot{v} = W_f - \Delta F_z \dot{v} \]

\[ F_{zr} = \frac{m g l_r}{l} + \frac{mh}{l} \dot{v} = W_r + \Delta F_z \dot{v} \]
# Tire/Road Interaction

<table>
<thead>
<tr>
<th>Year</th>
<th>Model Name</th>
<th>Model Properties</th>
<th>Features</th>
</tr>
</thead>
</table>
2. Has some revised formula |
| 1994 | Rill Model           | Semi-Empirical   | Easy to identify  
1. Can accurately fit curves  
2. Has lot of revised formula  
3. Can employ different factors |
2. Has lot of revised formula  
3. Can employ different factors |
2. Can produces smooth transition around zero velocity |
| 1991 | Bliman-Sorine Model  | Analytical       | Can capture the Strubeck effect in addition to Dahl model |
| 1995 | LuGre Model          | Analytical       | Can combine pre-sliding & sliding in addition to Bliman-Sorine Model |

The Tire/Road Interaction

The “Burkhardt model” describes the tire-road friction as a function of the wheel slip

\[ \mu(\lambda; \vartheta) = \vartheta_1 (1 - e^{-\lambda \vartheta_2}) - \lambda \vartheta_3 \]

By changing the values of the parameters \( \vartheta \), different road conditions can be modeled.

**Note:** the friction curve model, together with the fact that

\[ F_{x_i} = F_{z_i} \mu(\lambda_i; \vartheta) \]

implies that longitudinal forces are bounded \( |F_{x_i}| \leq \psi, \ i \in \{f, r\} \).

Due to tire relaxation dynamics, also the forces first time derivatives are bounded

\[ |\dot{F}_{x_i}| \leq \Gamma, \ i \in \{f, r\} \]
Braking Dynamics: slip dynamics

Wheel speed and wheel slip are linked by an algebraic relationship → can use the wheel slip as state variable for wheel dynamics

\[
\begin{align*}
\dot{\lambda}_f &= -\frac{r}{J_f} \left( \Psi_f(\lambda_f, \lambda_r) - T_{bf} \right) \\
\dot{\lambda}_r &= -\frac{r}{J_r} \left( \Psi_r(\lambda_f, \lambda_r) - T_{br} \right) \\
\dot{\psi} &= -\frac{W_f \mu(\lambda_f) + W_r \mu(\lambda_r)}{m - \Delta F_z(\mu(\lambda_f) - \mu(\lambda_r))}
\end{align*}
\]

The dependence of \(\Psi_r\) on \(\lambda_f\) can be easily managed within a SM framework → \(\Psi_r\) bounded for all \(\lambda_f\) and this is all one needs to know for designing a SM controller.

Two SISO controllers can be designed, and the coupling between front and rear wheel only affects the bounds on the uncertainties.

Antonella Ferrara 52
The selected **sliding variable** are the **slip tracking errors**

\[
s_i = \lambda_i - \lambda_i^*, \; i \in \{r, f\}
\]

**Auxiliary system**

\[
\begin{align*}
\dot{s}_i &= \dot{\lambda}_i - \dot{\lambda}_i^* \\
\ddot{s}_i &= \varphi_i + h_i \dot{\dot{\lambda}}_i
\end{align*}
\]

“auxiliary control signal”: it is the control law \(v(t)\) of the S-SOSM algorithm

\[
\varphi_i := \frac{r \omega_i}{v} + 2 \frac{r \omega_i \dot{v}}{v^2} - 2 \frac{r \omega_i v^2}{v^3} - \frac{r^2}{Jv} f_{xi} - \dot{\lambda}_i^*
\]

\[
h_i := \frac{r}{Jv}
\]

Both \(\varphi_i\) and \(h_i\) are fns of \(v\)!
S-SOSM Wheel Slip Controller (2)

Auxiliary system

\[
\begin{align*}
\dot{s}_i &= \lambda_i - \lambda_i^* \\
\ddot{s}_i &= \varphi_i + h_i \theta_i
\end{align*}
\]

“auxiliary control signal”

\[
|\varphi_i| \leq \Phi_i(v, \omega_i, T_{bi}) \\
0 < \Gamma_{i1}(v, \omega_i) \leq h_i \leq \Gamma_{i2}(v, \omega_i)
\]

Only the knowledge of the **upper-bounds** is necessary to design the control law.
Simulation Results

Results on a Simulink model including brakes dynamics, tire elasticity and tire relaxation (definitely more complex than that used for the design!)

Braking maneuver with set-point step-changes of width 0.05 from $\lambda^* = 0$ to $\lambda^* = 0.2$

Three algorithms compared: Suboptimal SOSM, FS-SOSM, GS-SOSM
Simulation Results (2)

\[ J_s = \sqrt{\frac{1}{N} \sum_{i=1}^{N} s(i)^2} \]

\[ J_{s\text{Norm}} = 100 \frac{J_s}{\max_{i=1,...,3} \{J_{s_i}\}} \]

The switched algorithms allow for a 30\% improvement with respect to the standard SOSM one.

The small difference between FS-SOSM and GS-SOSM probably due to the fact that \( J_s \) captures performance and both algorithms behave similarly in this respect.
Torque Vectoring Control in Fully Electric Vehicles via SMC
Torque Vectoring via SMC

• The torque-vectoring control of a four-wheel-drive fully electric vehicle with in-wheel drivetrains has been studied in recent years, and at ACC14 an ISM control based solution has been presented in collaboration with:

• The ISM controller is easy to tune and robust with respect to a significant set of uncertainties and disturbances.

• Its low complexity and other positive features make it appropriate for practical implementations.
European Project E-VECTOORC

- Vehicle Concept and Layout
- Powertrain Design and Safety
- Brake Design and EM-compatibility
- Vehicle Dynamics and Control
Electronic Stability Program (ESP) VS Torque-Vectoring (TV)

**ESP**
- Only when $|r_{ref} - r| > \theta$.
- ON-OFF control
- By means of:
  - Friction brakes
  - Wheel torques (-)

**TV**
- In any condition $(v, a_x, a_y, \delta)$
- At any instant of time
- By means of:
  - Friction brakes
  - Wheel torques (+/-)
Why Sliding Mode for Yaw Rate Control?

• Conventional yaw rate control systems: PID + FF
  – Good tracking performance and large bandwidth of the closed-loop system
  – Non robust and smooth enough in critical conditions (high lateral acceleration)

• Many other approaches have been investigated (some examples):
  – Internal Model Control;
  – MPC/Linear Quadratic Control;
  – Optimal Control based on LMI.

• Major motivations for using Integral Sliding Mode Control:
  – Ease of implementation (few parameters only);
  – High level of robustness against unmodeled dynamics and external disturbances, even when implemented with sampling times typical of real automotive control applications (experimental tests support this claim).
The Vehicle Model

The proposed controller has been designed considering the vehicle yaw dynamics

\[ x = f(x) + B(x)u + h(x,t) \]

\[ rJ_z = \left( -F_{y,RF} \sin \delta_{RF} + F_{y,LF} \sin \delta_{LF} + F_{x,RF} \cos \delta_{RF} - F_{x,LF} \cos \delta_{LF} \right) \cdot \frac{T_f}{2} + \]

\[ \left( F_{y,RF} \cos \delta_{RF} + F_{y,LF} \cos \delta_{LF} + F_{x,RF} \sin \delta_{RF} + F_{x,LF} \sin \delta_{LF} \right) \cdot a + \]

\[ \left( F_{y,RR} + F_{y,LR} \right) \cdot b + \left( F_{x,RR} - F_{x,LR} \right) \cdot \frac{T_r}{2} - M_{z,RF} - M_{z,LF} - M_{z,RR} - M_{z,LR} \]
Design of the proposed ISM controller

In the previous model, the overall yaw moment is the control variable

\[
\cdot \cdot \cdot \cdot 
- r_{\text{err}} = r - r_{\text{ref}} = k(r, \beta, \delta) \cdot r_{\text{ref}} + \frac{1}{J_z} M_{z,\text{ISM}} + \frac{1}{J_z} M_{z,\text{dist}}
\]

Unknown part: \( h = k(r, \beta, \delta) + \frac{1}{J_z} M_{z,\text{dist}} \)

Known part: \( - r_{\text{ref}} \)

\[
k(r, \beta, \delta) = \frac{1}{J_z} \left( - F_{y,RF} \sin \delta_{RF} + F_{y,LF} \sin \delta_{LF} \right) \frac{T_f}{2} + \left( F_{y,RF} \cos \delta_{RF} + F_{y,LF} \cos \delta_{LF} \right) \cdot a - \left( F_{y,RR} + F_{y,LR} \right) \cdot b - M_{z,RF} - M_{z,LF} - M_{z,RR} - M_{z,LR}
\]
The Yaw Rate Control Scheme

Wheel torques due to driver’s intention

Demanded Wheel Motor Torque

Pedal brake pressure

Torque and Yaw Moment

Drivability Controller

Reference Yaw Rate Generator

High-Level Controller

Control Allocation

Vehicle Plant

\[ \frac{a_{pp}}{b_{pp}} \]

\[ T_{w,\text{tot}} \]

\[ r_{\text{ref}} \]

\[ r_{\text{err}} \]

\[ \delta, \nu, a_x, \mu \]

\[ T^*, M_{z,\text{ISM}} \]

\[ T_{m,i} \]

\[ p_{b,i} \]

\[ \delta, \nu, a_x, \mu \]

\[ r \]

Measured/estimated quantities
Integral Sliding Mode Control

\[ M_{z,SM} = -J_z K_{SM} \text{sign}(s) \]

\[ \tau_{SM} \dot{M}_{z,SM,fil} + M_{z,SM,fil} = M_{z,SM} \]

\[ S = S_0 + Z \]

Where

\[ \dot{z} = -\frac{\partial S_0}{\partial (-r_{err})} \left[-r_{ref} + \frac{1}{J_z} (M_{z,ISM} - M_{z,SM}) \right] \]

with the initial condition \( z(0) = -S_0(r_{err}(0)) \)

The sliding mode starts from the first sampling instant, without any reaching-phase transient.
**Controller Parameters Design**

Only two parameters needs to be tuned: $K_{SM}$ and $\tau_{SM}$

- **$\eta$-reaching condition** $s\dot{s} < -\eta|s|$ fulfilled for any value $K_{SM} > J_z \cdot |h_{\text{max}} + \eta|$

- A sequence of step steer maneuvers at $v=90 \text{ km/h}$ (high critical situation) has provided the upper bound $J_z \cdot |h_{\text{max}}| \approx 9,000 Nm$

- A conservative value has been chosen: $K_{SM} = 15,000 Nm$

- The value $\tau_{SM} = 0.05s$ has been selected in order to avoid the **chattering effect**, so that the yaw moment generated by the high-level controller does not induce vibrations perceivable by the passengers.
The Show Case
The Show Case

• The performance assessment has been carried out using an accurate IPG CarMaker model of the vehicle (validated on the basis of experimental tests at the Lommel Proving Ground in Belgium)

• Two conventional test maneuvers have been considered:
  - Ramp Steer Maneuver
  - Step Steer Maneuver

• The controlled vehicle ("Sport Mode") has been compared with the passive vehicle ("Baseline Mode")
Ramp Steer Maneuver (I)

Benefits of the adoption of the ISM yaw rate controller:

- lower value of the understeer gradient;
- extension of the linear region of the vehicle understeer characteristic;
- higher values of lateral acceleration.
Reference yaw rate tracking performance at different speeds.

The reference (dashed dark line) and the yaw rate of the controlled vehicle (blue line) are practically overlapped. The red line is the yaw rate of the passive vehicle.
Step Steer Maneuver

- Evident degradation of the passive vehicle tracking performance at increasing speeds
- Good tracking of the yaw rate reference exhibited by the controlled vehicle, with very short rise-time and negligible overshoot.

- Obtained by means of a control signal perfectly acceptable in automotive applications.
Summary Results

\[ RMSE = \sqrt{\frac{1}{t_{\text{end}} - t_{\text{in}}} \int_{t_{\text{in}}}^{t_{\text{end}}} (r_{\text{err}})^2 \, dt} \]

**BLUE**: Controlled veh.  **RED**: Passive veh.
Some Final Remarks

• Sliding mode control is an interesting methodology for its simplicity and robustness features, but it may be non appropriate to be applied, in its conventional formulation, to the automotive context due to the discontinuous control variable.

• The second order sliding mode controller generates continuous control actions, thus limiting the vibrations it can induce and propagate throughout the vehicle subsystems because of chattering effects.

• The switched version of Suboptimal SOSM control law allows to tune the controller parameters taking into account the vehicle speed. For this reason it is particularly appropriate for motorbikes, the slip dynamics of which strictly depends on speed.

• Integral SMC can also be an effective solution.
References


References


Other topics in the automotive field:


MARIE SKŁODOWSKA-CURIE ACTIONS
Innovative Training Networks (ITN) Call: H2020-MSCA-ITN-2015

Interdisciplinary Training Network in Multi-Actuated Ground Vehicles
“ITEAM”

1 Ph.D. Position possibly available at University of Pavia since June 1st, 2016!
Please contact Prof. Antonella Ferrara since October 1st, 2015 for application informations