Spring School 2015

Slding-Mode Control _ Theory and Applications

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Lecture on Optimization-based High Order Sliding Mode Control and Applications. Part 4: Automotive Applications

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Summary

- **Traction Control with Fastest Acceleration/Deceleration via Second Order Sliding Mode Control**

- **Vehicles Platooning via Second Order Sliding Mode Control**

- **Braking Control of Two-Wheeled Vehicles via Switched Second Order Sliding Mode Control**

- **Torque Vectoring Control in Fully Electric Vehicles via SMC**

Traction Control with Fastest Acceleration/Deceleration Via Second Order Sliding Mode Control

Traction Force Control

- The **traction force control** is an automatic driver assistance system which increases **vehicle drivability** expecially in difficult wheater conditions, allowing **anti-skid braking** and **anti-spin acceleration**.

- The traction force produced by a vehicle is strongly influenced by **road conditions**.

- It is necessary to design a **robust traction force controller** taking into account the time-varying tire/road interaction.

- As a further requirement, the designed control law has to prevent the generation of **vibrations** which could increase the **discomfort** index.

The Vehicle Model

$$m\dot{v}_x = \left\lfloor F_{xf}\left(\lambda_f\right) + F_{xr}\left(\lambda_r\right)\right\rfloor - F_{loss}$$

$$F_{loss} = F_{air}\left(v_x\right) + F_{roll}$$

$$J_i\dot{\omega}_i = T_i - r_i F_{xi}\left(\lambda_i\right)$$

$$f_i = -\frac{\dot{v}_x}{\omega_i r_i} - \frac{v_x}{J_i\omega_i^2}F_{xi}$$

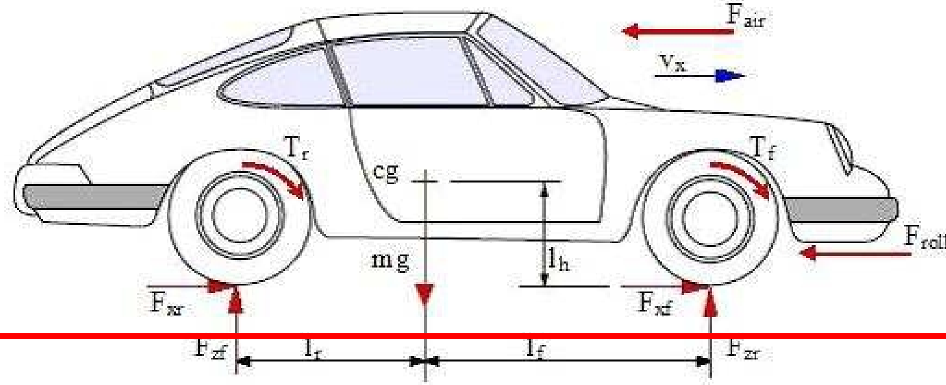
$$\dot{\lambda}_i = f_i + h_i(v_x)T_i$$

$$i \in \{f, r\}$$

$$h_i(v_x) = \frac{v_x r_i}{J_i\left(\omega_i r_i\right)^2}$$

$$F_{zf} = \frac{l_r mg - l_h m\dot{v}_x}{2(l_f + l_h)}$$

$$F_{zr} = \frac{l_f mg + l_h m\dot{v}_x}{2(l_f + l_h)}$$



- Bicycle model (*"single-track model"*)
- Roll and yaw dynamics, lateral and vertical motions, and actuators dynamics are neglected
- To design the controller, the normal forces are assumed to be constant (this assumption is removed in simulations)

The Traction Force Control Problem

- The traction force F_x depends on the **tire/road interaction**, (which depends on the road conditions), on the the normal force F_z, and on the **"slip ratio"** λ

$$\boxed{\lambda := \frac{\omega r - v_x}{\max\left(\omega r, v_x\right)}}$$

- The **"traction controller"** can be realized as a **"slip controller"**: to control λ so that the desired traction force is produced, by using the torques acting on the front and rear wheel, T_f, T_r, as control variables.

- To design the slip controller it is necessary to model the **tire/road interaction**.

Tire/Road Interaction Models

Year	Model Name	Model Properties	Features
	Piecewise Linear Model	Empirical	1. Cannot accurately fit curves 2. Easy to identify
1993	Burckhardt Model	Semi-Empirical	1. Can accurately fit curves 2. Has some revised formula
1994	Rill Model	Semi-Empirical	Easy to identify
1987	Magic Formula	Semi-Empirical	1. Can accurately fit curves 2. Has lot of revised formula 3. Can employ different factors
1977	Dahl Model	Analytical	1. Can describe Coulomb friction. 2. Can produces smooth transition around zero velocity
1991	Bliman-Sorine Model	Analytical	Can capture the Stribeck effect in addition to Dahl model
1995	LuGre Model	Analytical	Can combine pre-sliding & sliding in addition to Bliman-Sorine Model

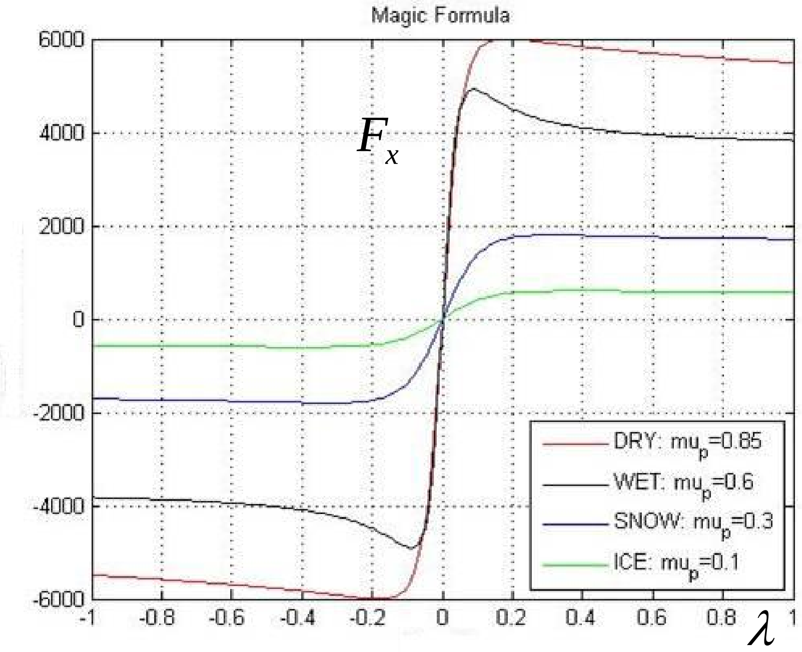
Table from: ***Li Li, Fei-Yue Wang, and Qunzhi Zhou, "Integrated Longitudinal and Lateral Tire/Road Friction Modeling and Monitoring for Vehicle Motion Control", IEEE Trans. On Intelligent Transportation Systems, Vol. 7, No. 1, March 2006.***

The Tire/Road Interaction

The well-known **"Magic Formula"** by Bakker-Pacejka models the traction force as a function of the slip ratio

$$\boxed{F_x = \mu_p \cdot f_t(\lambda, F_z)}$$

parametrized by the **tire-road adhesion coefficient** μ_p

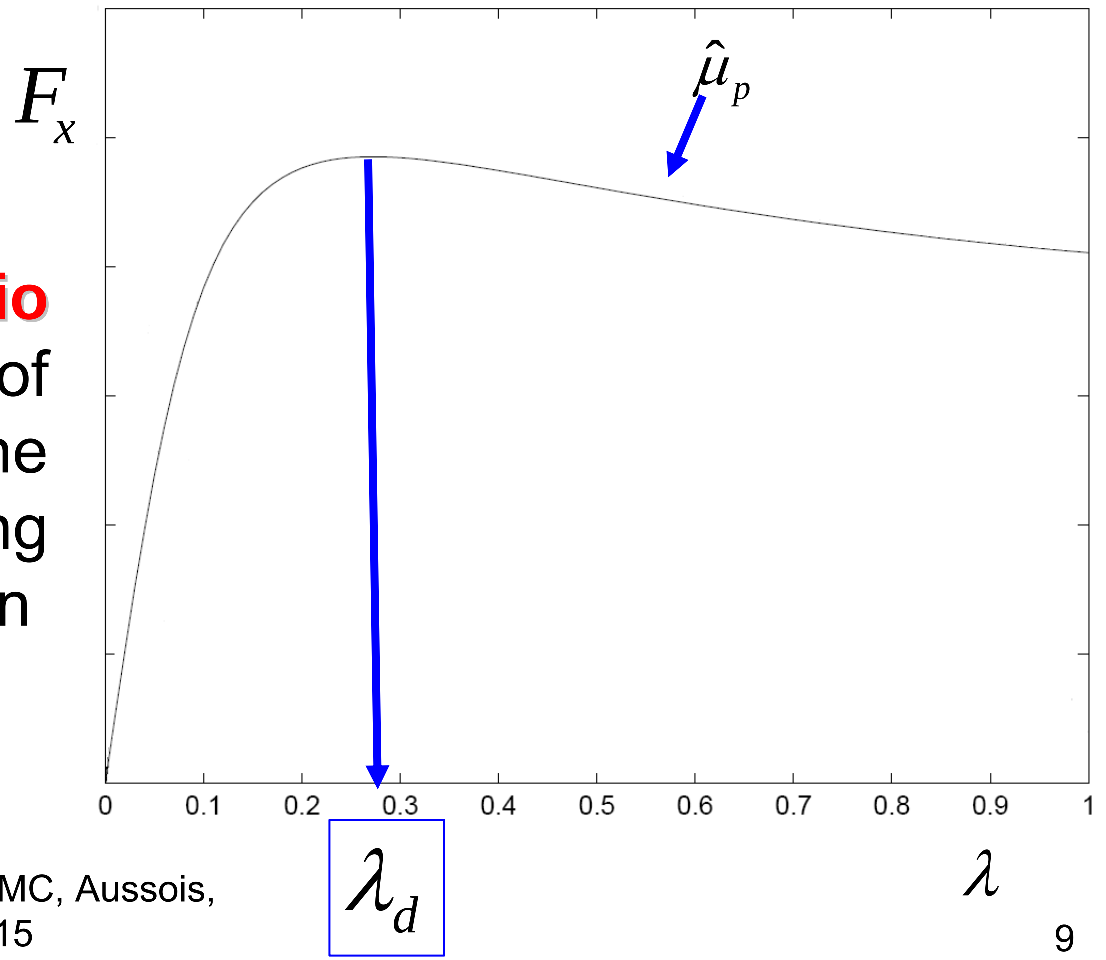


μ_p depends on road conditions and needs to be estimated

The Fastest Acceleration/Deceleration Control (FADC) Problem

The control objective for the **FADC problem** is to maximize the generated traction force

The **desired slip ratio** λ_d is the abscissa of the extremal value of the $\lambda - F_x$ curve corresponding to the actual road condition



Solution of the FADC Problem

To solve the **FADC problem** the control scheme must accomplish the following tasks

1. **To estimate on-line the adhesion coefficient** μ_p to identify the actual $\lambda - F_x$ curve

2. **To calcuate the desired slip ratio** λ_d as the abscissa of the extremal value of the actual $\lambda - F_x$ curve

3. **To design a control law** that makes the actual slip ratio λ track the desired value λ_d

The Adhesion Coefficient Estimate 1/2

The adhesion coefficient can estimated using a **recursive least square (RLS)** technique with "forgetting factor".

The objective is to find the value $\hat{\mu}_p$ that minimizes the following cost

$$\boxed{J = \int_0^t e^{-\int_\tau^t \rho(r)dr} \left\| y(\tau) - \hat{\mu}_p(t)\phi(\tau) \right\|^2 d\tau}$$

$\rho(t) \geq 0$ forgetting factor

$$y(t) = \frac{1}{2}\left(m\dot{v}_x(t) - F_{loss}(t)\right) = \mu_p(t)\phi(t)$$

measurable quantities

$$\phi(t) = f_{t_f}(t) + f_{t_r}(t)$$

The Adhesion Coefficient Estimate 2/2

The **parameter update law** is given by

$$\dot{\hat{\mu}}_p = -P(t)\phi(t)e(t)$$

where $e(t) = \hat{\mu}_p\phi(t) - y(t)$ and $P(t)$ is the **update gain** of parameter $\hat{\mu}_p$

The **update gain** can for instance be adjusted as

$$\dot{P}(t) = \rho(t)P(t) - \phi(t)^2 P(t)^2$$

The Slip Controller 1/2

The selected **sliding variables** are the **"slip tracking error"** at the front and rear wheel, respectively

$$\boxed{s_i = \lambda_i - \lambda_{i_d} = \sigma_{i1}} \qquad i \in \{f, r\}$$

Auxiliary system

$$\begin{cases} \dot{s}_i = \dot{\sigma}_{i1} = \sigma_{i2} & i \in \{f, r\} \\ \ddot{s}_i = \dot{\sigma}_{i2} = \overbrace{\dot{f}_i + \dot{h}_i T_i - \ddot{\lambda}_{i_d}}^{\varphi_i(t)} + \overbrace{h_i}^{\gamma_i(t)} \dot{T}_i \end{cases}$$

"auxiliary control"

unmeasurable

$$|\varphi_i(t)| < \Phi_i \qquad 0 < \Gamma_{i1} \le \gamma_i(t) \le \Gamma_{i2}$$

only the knowledge of the **upperbounds** is necessary to design the control law

The Slip Controller 2/2

The **auxiliary systems** are in a form suitable to apply a **second order sliding mode techinque**

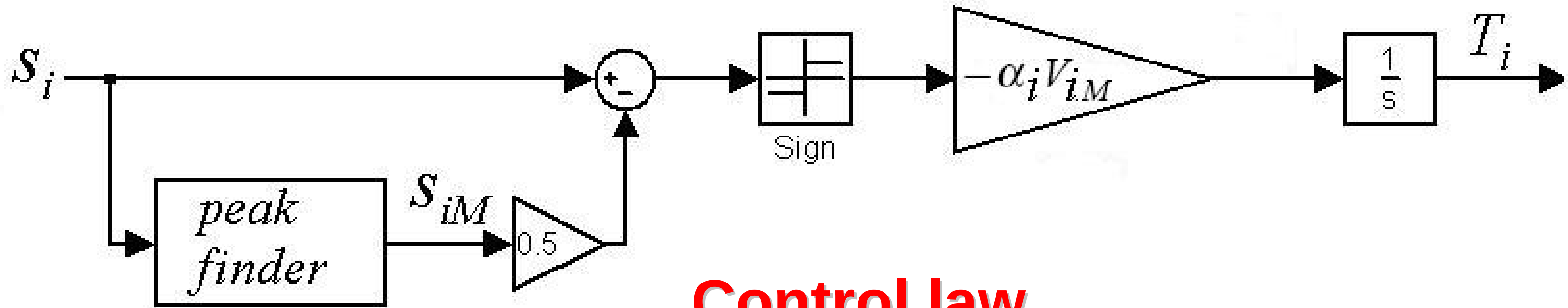
so that, the sliding variables and their first time derivatives,

$\sigma_{i1} = s_i \quad \sigma_{i2} = \dot{s}_i$, are steered to zero in a finite time.

⇒ As a consequence, the **slip error** relevant to the **front and rear wheel** vanishes in a finite time

$$\boxed{s_i = \lambda_i - \lambda_{i_d} = 0}$$

The Sub-Optimal Controller



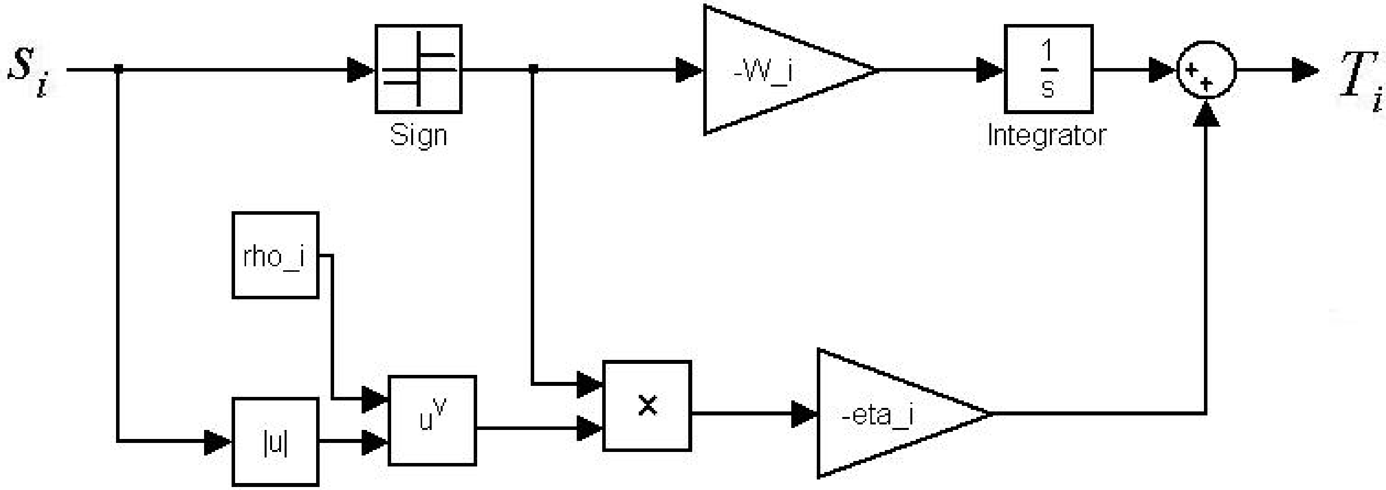
Control law

$$\dot{T}_i(t) = -\alpha_i V_{iM}\,\mathrm{sgn}\left(s_i(t) - \frac{1}{2} s_{iM} \right), \qquad i \in \{f, r\}$$

$$V_{iM} > \max\left(\frac{\Phi_i}{\alpha_i^* \Gamma_{i1}} ; \frac{4\Phi_i}{3\Gamma_{i1} - \alpha_i^* \Gamma_{i2}} \right) \qquad \alpha_i^* \in (0,1] \cap \left(0, \frac{3\Gamma_{i1}}{\Gamma_{i2}} \right)$$

$$\alpha_i = \begin{cases} \alpha_i^* & [x_{i1}(t) - 0.5 x_{iM}] \cdot [x_{iM} - x_{i1}(t)] > 0 \\ 1 & \text{otherwise} \end{cases}$$

The Super Twisting Controller



Control law

$$T_i(t) = -\eta_i |s_i|^{\rho_i} \operatorname{sgn}(s_i) + u_{1i}(t), \quad i \in \{f, r\}$$

$$\dot{u}_{1i}(t) = -W_i \operatorname{sgn}(s_i)$$

$$0 < \rho_i \le 0.5 \qquad W_i > \frac{\Phi_i}{\Gamma_{i1}} \qquad \eta_i^2 > \frac{4\Phi_i \Gamma_{i2}(W_i + \Phi_i)}{\Gamma_{i1}^3 (W_i - \Phi_i)}$$

Simulation Test

In the simulated scenario, the vehicle performs an acceleration manoeuvre for 6 *s* starting from the velocity of *20 m/s*. The road condition changes at 3 *s* from dry asphalt to wet asphalt

g	9.81	m/s^2
m	1366	kg
J_{wf}	1.07	kgm^2
J_{wr}	1.07	kgm^2
c_x	0.4	
f_{roll}	0.013	
r_{wf}	0.32	m
r_{wr}	0.32	m
$v_x(0)$	30	m/s
$\lambda_f(0)$	0.15	
$\lambda_r(0)$	0.15	
$V_{fM} = V_{rM}$	2000	N/ms
$\alpha_f^* = \alpha_r^*$	1	
$\eta_f = \eta_r$	100	N/ms
$W_f = W_r$	3000	N/ms^2
$\rho_f = \rho_r$	0.5	

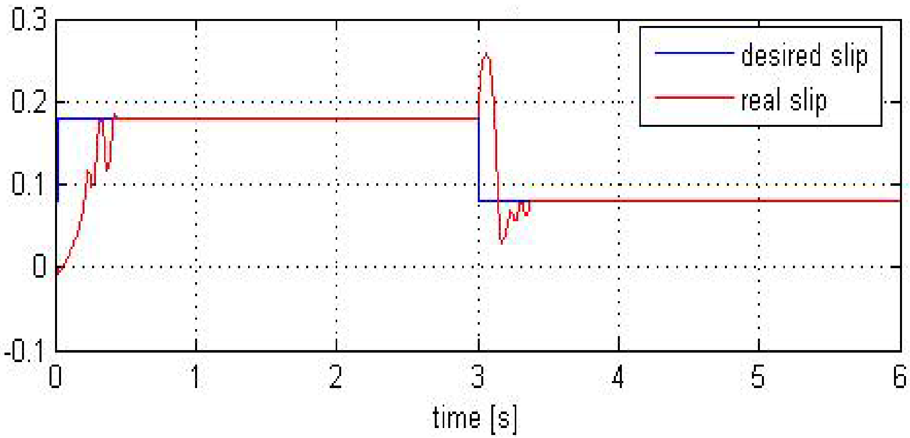
The **total available braking torque** and the **engine torque** exerted on the driving shaft can be determined from $T_j, \quad j \in \{f, r\}$

For instance, for a front-wheel-driven (FWD) vehicle:

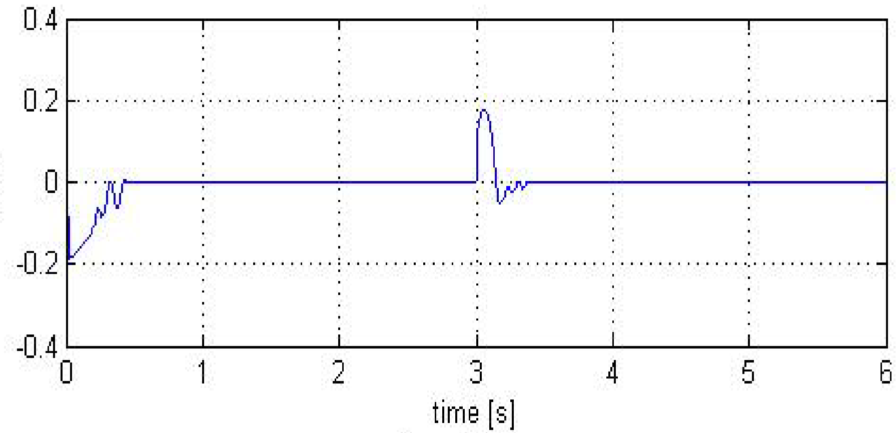
$$\boxed{\begin{aligned} T_f &= 0.5T_{shaft} - 0.3T_{brake} \\ T_r &= -0.2T_{brake} \end{aligned}}$$

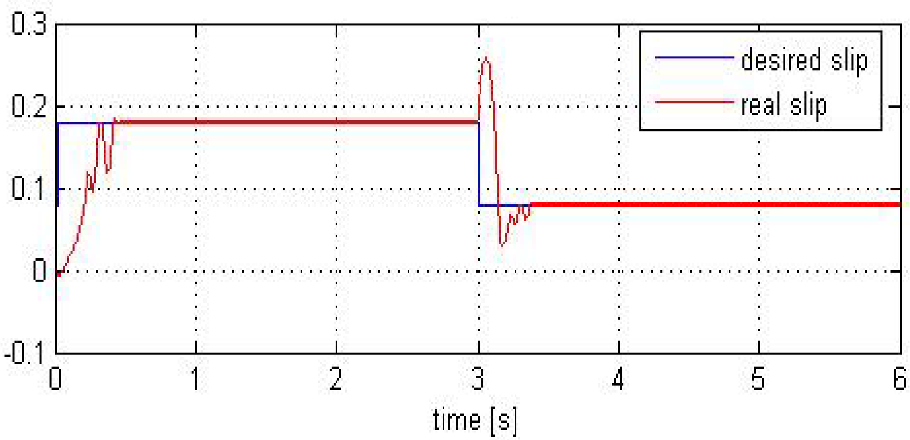
Sub-Optimal: Simulation Results 1/2

Slip at the front wheel

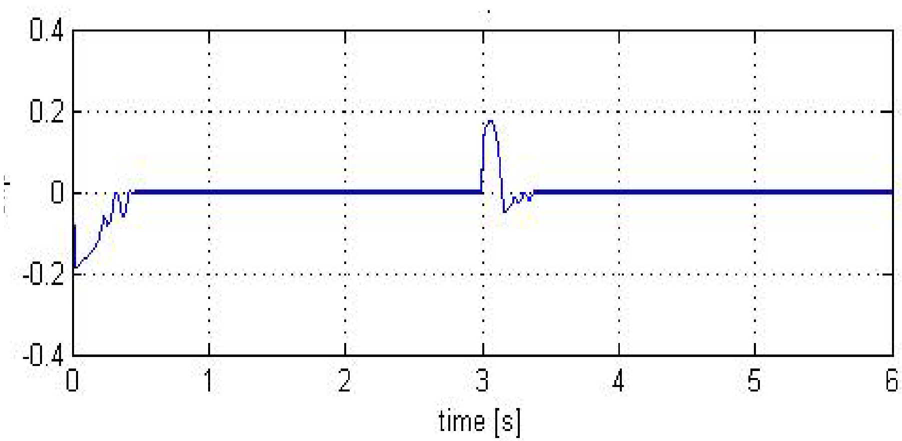


Slip error at the front wheel





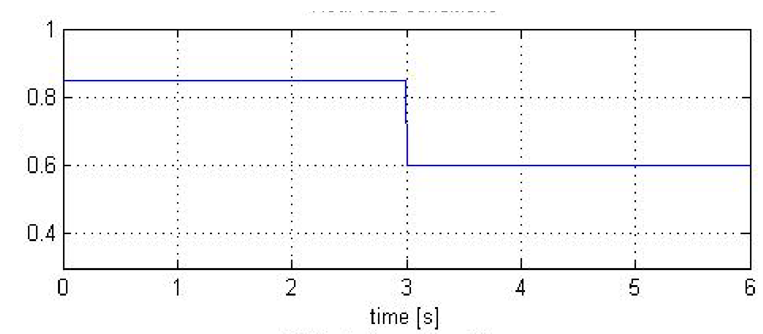
Slip at the rear wheel



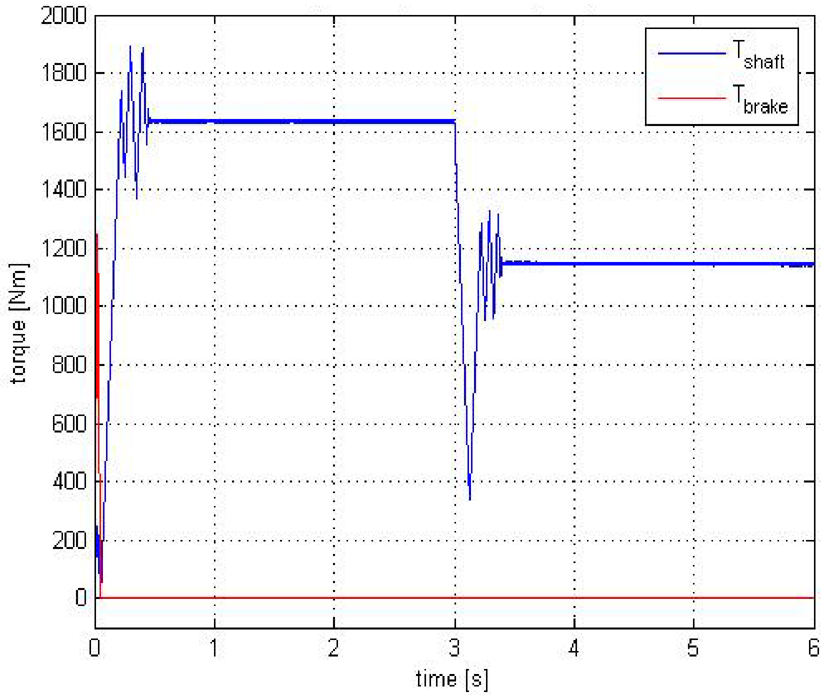
Slip error at the rear wheel

Sub-Optimal: Simulation Results 2/2

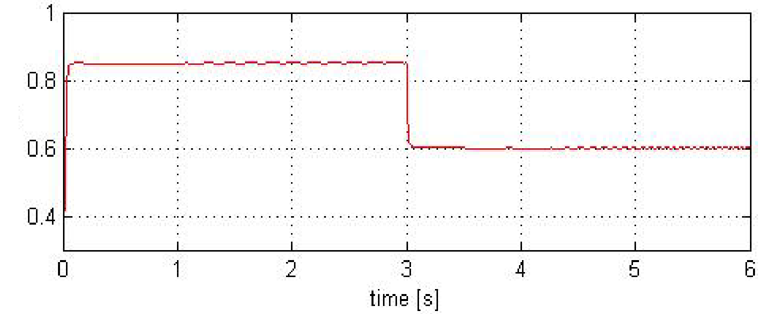
Real road condition



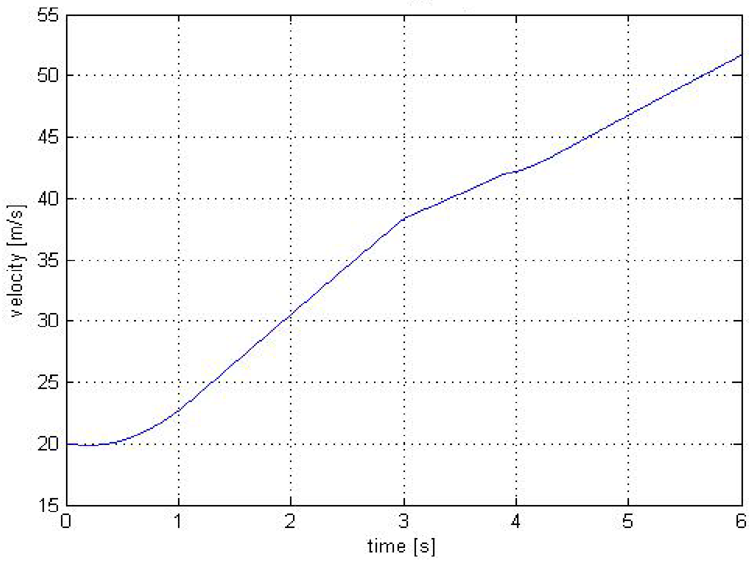
Engine and Braking Torque



Estimated road condition

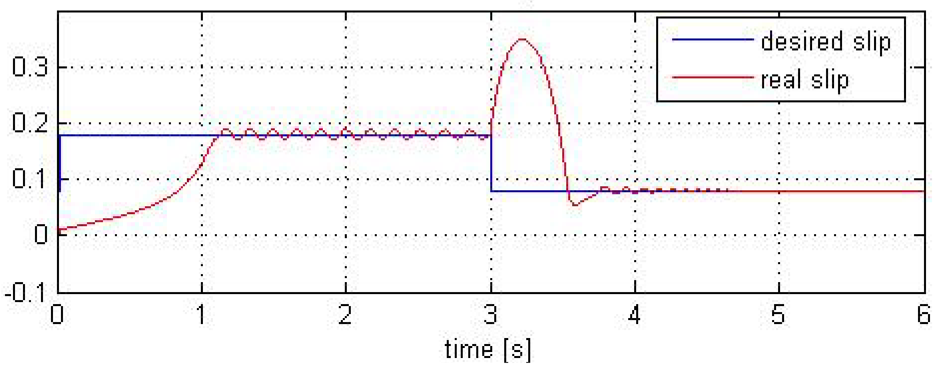


Vehicle Velocity

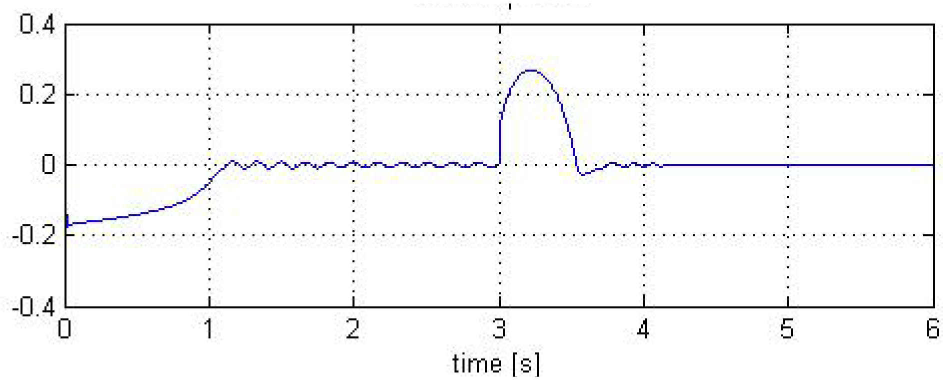


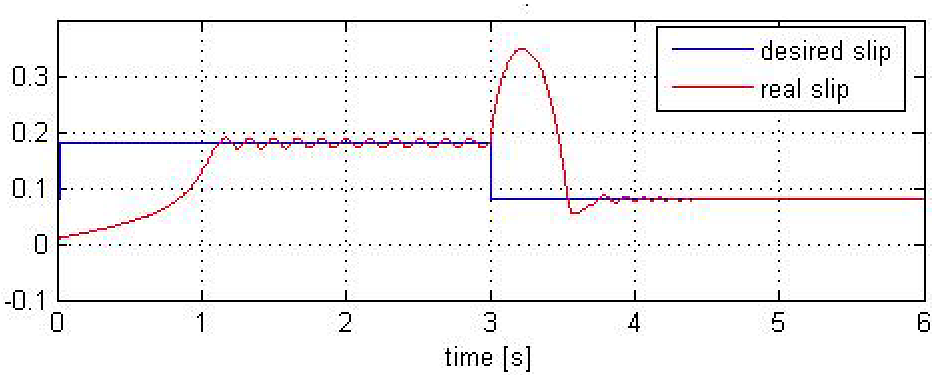
Super Twisting: Simulation Results 1/2

Slip at the front wheel

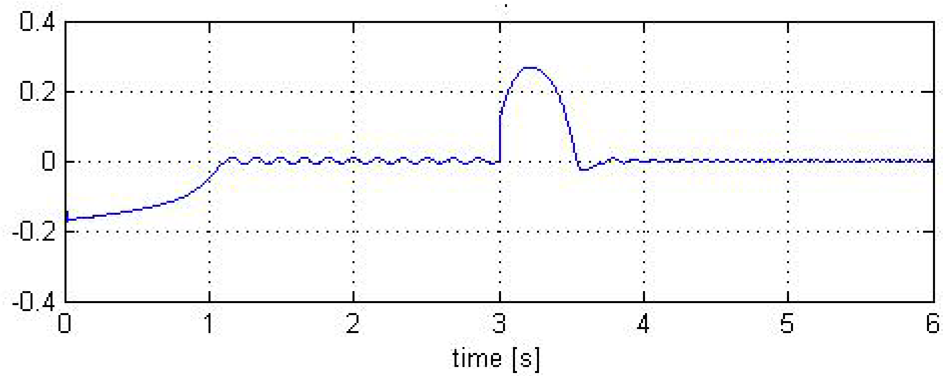


Slip error at the front wheel





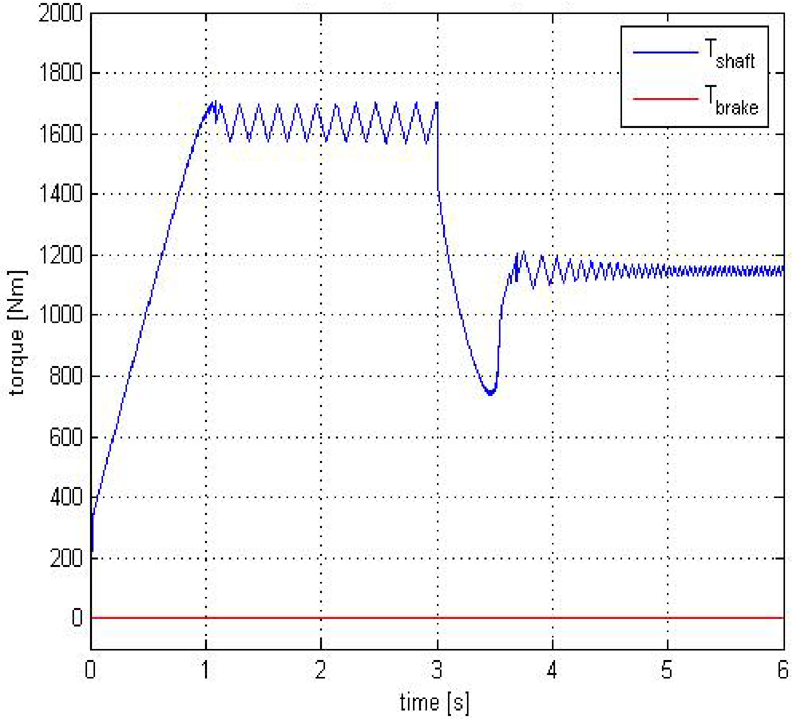
Slip at the rear wheel



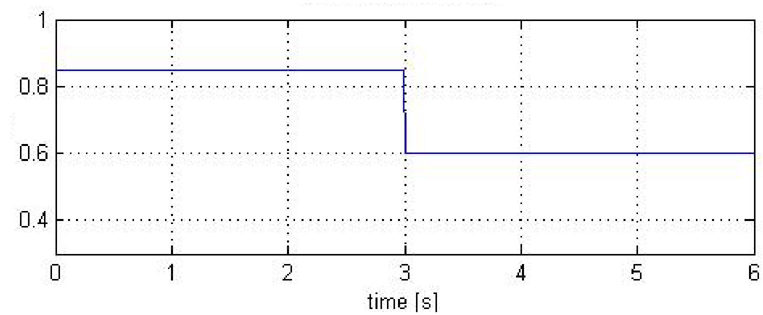
Slip error at the rear wheel

Super Twisting: Simulation Results 2/2

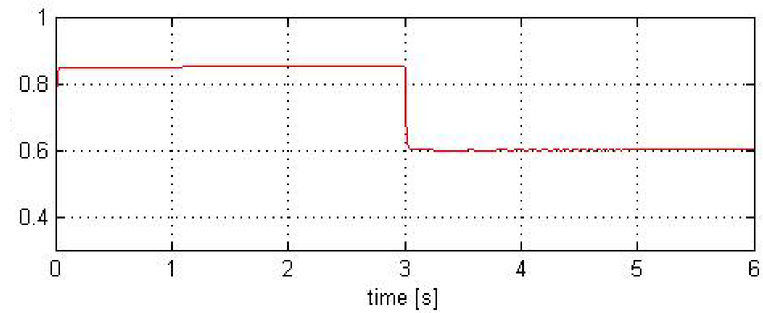
Engine and Braking Torque



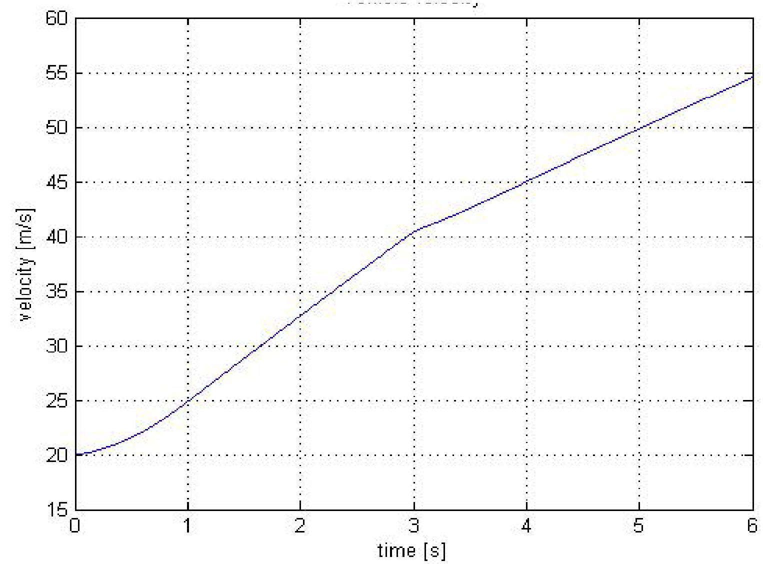
Real road condition



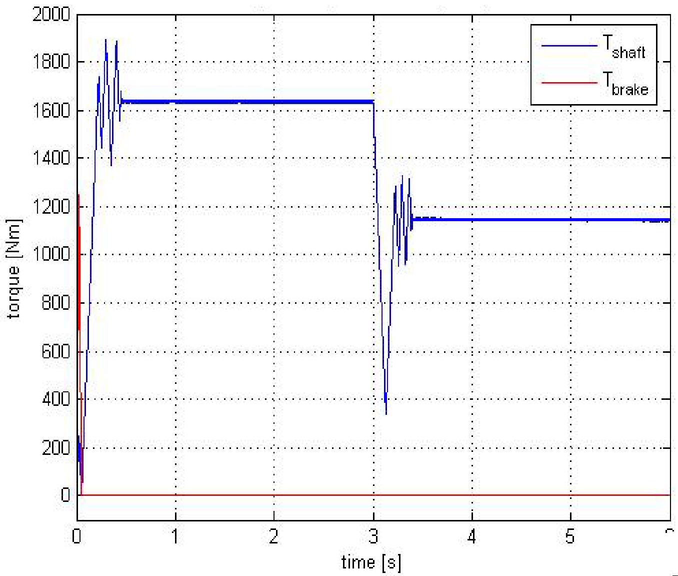
Estimated road condition



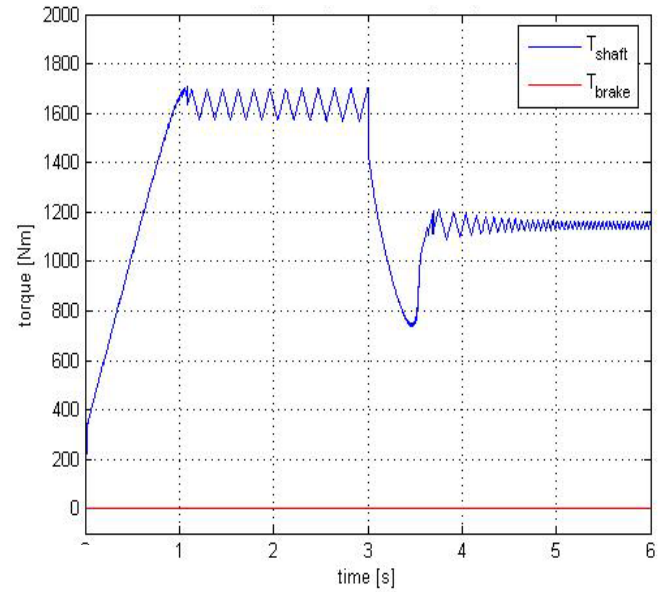
Vehicle Velocity



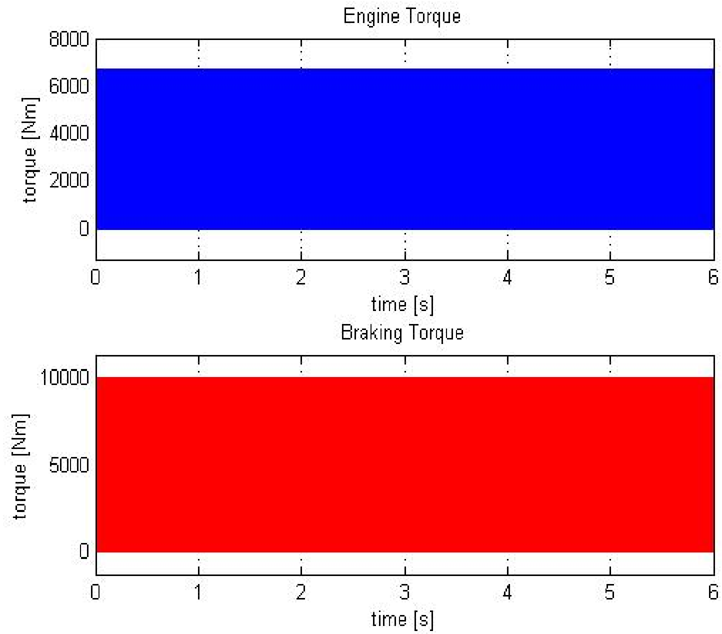
Simulation Results: A Comparison



Braking torque and engine torque with the considered second order sliding mode controllers



Braking torque and engine torque with a conventional first order sliding mode controller



Vehicles Platooning via Second Order Sliding Mode Control

Control of a Platoon of Vehicles

- The longitudinal control of platoons of vehicles is effective to improve **traffic capacity** and **road safety** by reducing traffic dishomogeneities.

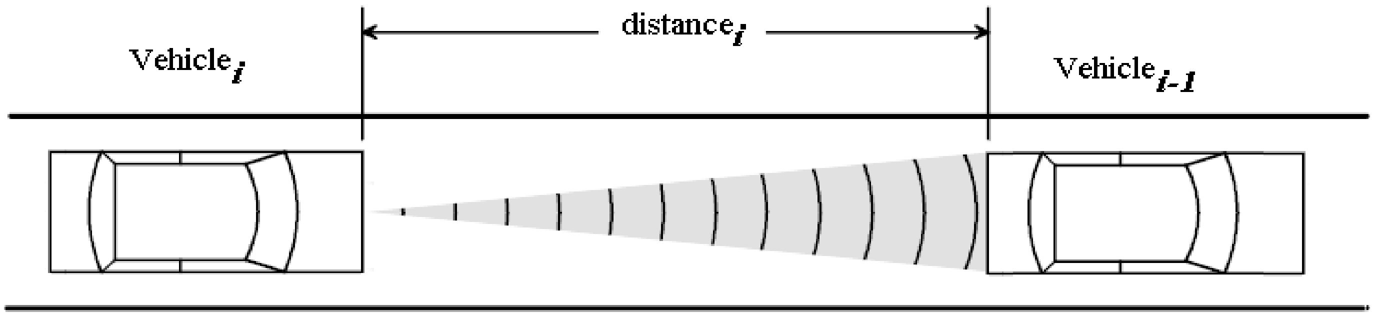
- As for traction control, the performances of a **“platooning control system”** are strongly influenced by **road conditions**.

- The **“platooning control problem”** has to be solved in a robust way.

- As a further requirement, also in this case, the designed control law has to prevent the generation of **vibrations** induced by the controller.

The Platooning Control Problem 1/2

- **Control objective:** make the distance between two subsequent vehicles of the platoon be equal to a pre-specified **"safety distance"** .



$$d_i = x_{i-1} - x_i = x_{spacing}\left(v_{x_i}\right)$$

- Solving the **platooning problem** even in presence of possible changes of the road conditions requires to re-formulate the problem as a **traction control problem** and take again into account the **tire/road interaction**.

The Platooning Control Problem 2/2

A possible way to determine the **"safety distance"** is to rely on the following **"variable spacing policy"**

$$\boxed{x_{spacing}\left(v_{x_i}\right) = S_{d_0} + h v_{x_i}}$$

follower's velocity

distance between stopped vehicles

headway time

To solve the platooning control problem as a traction control problem means to design a controller capable of generating the traction force F_x suitable to steer to zero the **"spacing error"**

$$\boxed{e_i = x_{spacing}\left(v_{x_i}\right) - d_i}$$

The Traction Control Problem

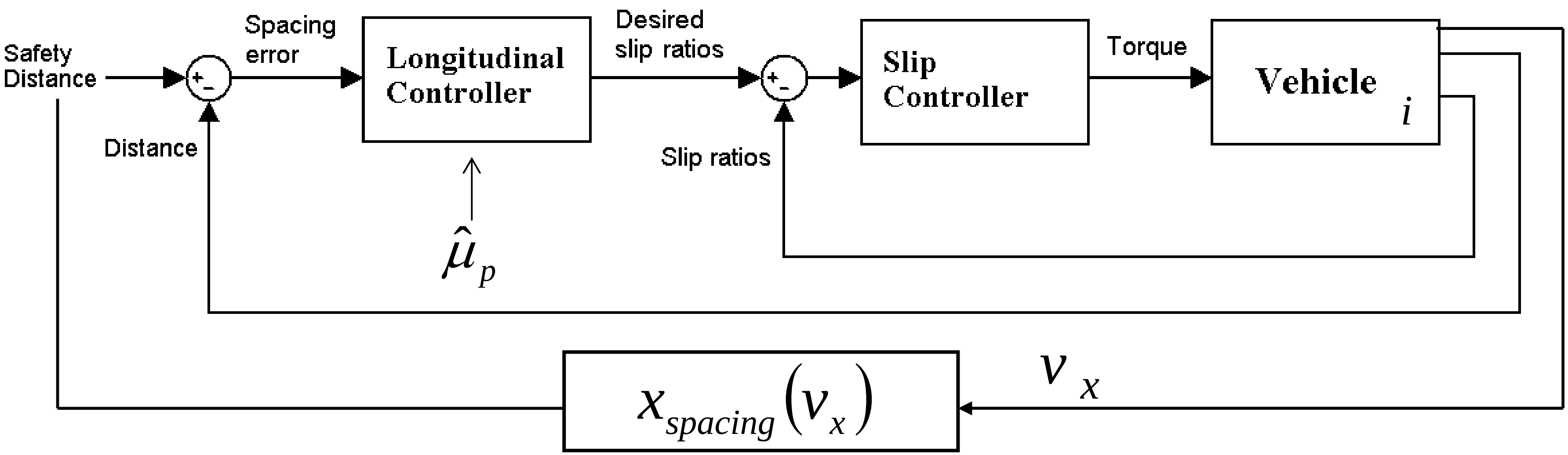
- The traction force F_x depends on the **tire/road interaction**, (which depends on the road conditions), on the the normal force F_z, and on the **"slip ratio"** λ

$$\boxed{\lambda := \frac{\omega r - v_x}{\max\left(\omega r, v_x\right)}}$$

- The **"traction control"** can be realized as a **"slip control"**: to control λ so that the desired traction force is produced, by using the torques acting on the front and rear wheel, T_f, T_r.

- To design the slip control it is necessary to model the **tire/road interaction**. Also in this case the Bakker-Pacejka model can be used.

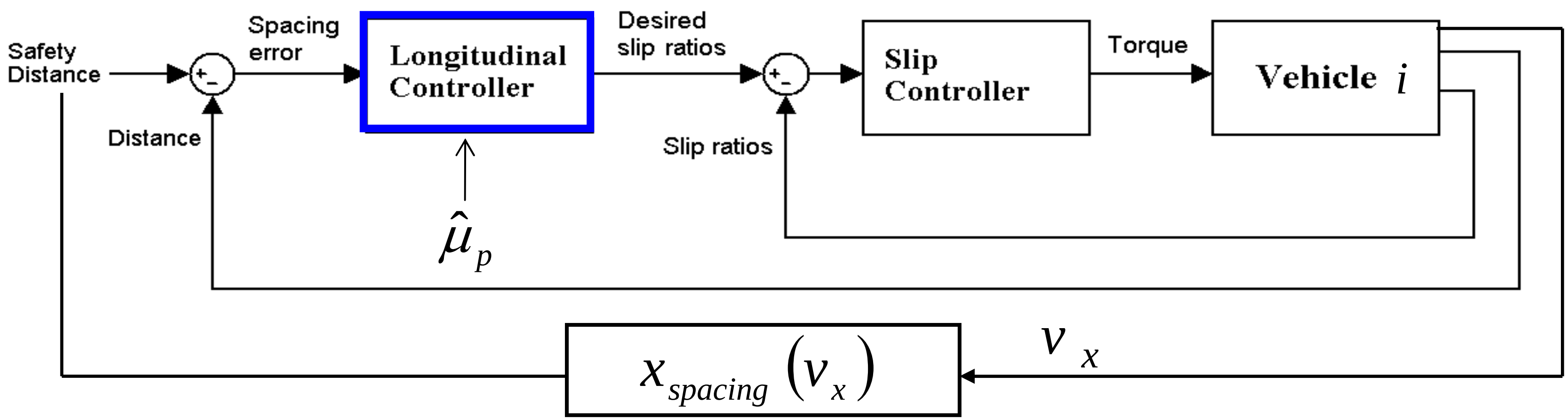
The Control Scheme



Outer loop: on the basis of the **spacing error** the controller determines the **desired traction force**. Relying on this latter and on the current tire/road adhesion coefficient the **desired slip ratios** are calculated. These are references for the inner loop.

Inner loop: has the objective of attaining the desired slip ratios by acting on the **torques at the front and rear wheels**.

The Longitudinal Controller Design 1/4



Sliding variable: the **"spacing error"** associated with the *i*-th vehicle

$$\boxed{S_i = e_i = S_{d_0} + hv_{x\,i} - x_{i-1} + x_i}$$

To design a **"second order sliding mode control law"**, determine the first and second time derivatives

$$\dot{S}_i = \dot{e}_i = v_{x\,i} - v_{x\,i-1} + h\dot{v}_{x\,i}$$

$$\ddot{S}_i = \ddot{e}_i = \dot{v}_{x\,i} - \dot{v}_{x\,i-1} + h\ddot{v}_{x\,i}$$

The Longitudinal Controller Design 2/4

By introducing the **auxiliary variables**

$$y_{1_i} = S_i \qquad y_{2_i} = \dot{S}_i$$

the previous differential equations can be rewritten as

$$\begin{cases} \dot{y}_{1_i} = y_{2_i} \\ \dot{y}_{2_i} = \varepsilon_i + w_i \end{cases}$$

unmeasurable (ε_i)

"auxiliary control" (w_i)

where $\left|\varepsilon_i\right| = \left| \dot{v}_{xi} - \dot{v}_{xi-1} \right| \leq \Gamma_i$ and $w_i = h\ddot{v}_{xi}$

→ only the knowledge of the **upperbound** is necessary to design the control law

The Longitudinal Controller Design 3/4

It can be proved that, if the **auxiliary control signal** is chosen (for instance) as

$$w_i(t) = -W_{M_i}\, sign\left\{ y_{1_i} - \frac{1}{2} y_{1_i MAX} \right\}, \quad y_{1_i MAX} = y_{1_i}\Big|_{y_{2_i}=0}$$

where $W_{M_i} > 2\Gamma_i$

then, the sliding variable and its first time derivative,

$y_{1_i} = S_i \quad y_{2_i} = \dot{S}_i$, are steered to zero in a finite time.

As a consequence, the spacing error between two subsequent vehicles vanishes in a finite time

$$S_i = e_i = S_{d_0} + hv_{x\,i} - x_{i-1} + x_i = 0$$

The Longitudinal Controller Design 4/4

On the basis of the auxiliary signal $w_i(t)$, the desired traction force (the **actual control signal !**) can be determined as

$$\boxed{F_{xf_{i_{des}}} + F_{xr_{i_{des}}} = \frac{1}{2}\left(m_i \dot{v}_{x_{i_{des}}} + F_{loss\ i}\right)}$$

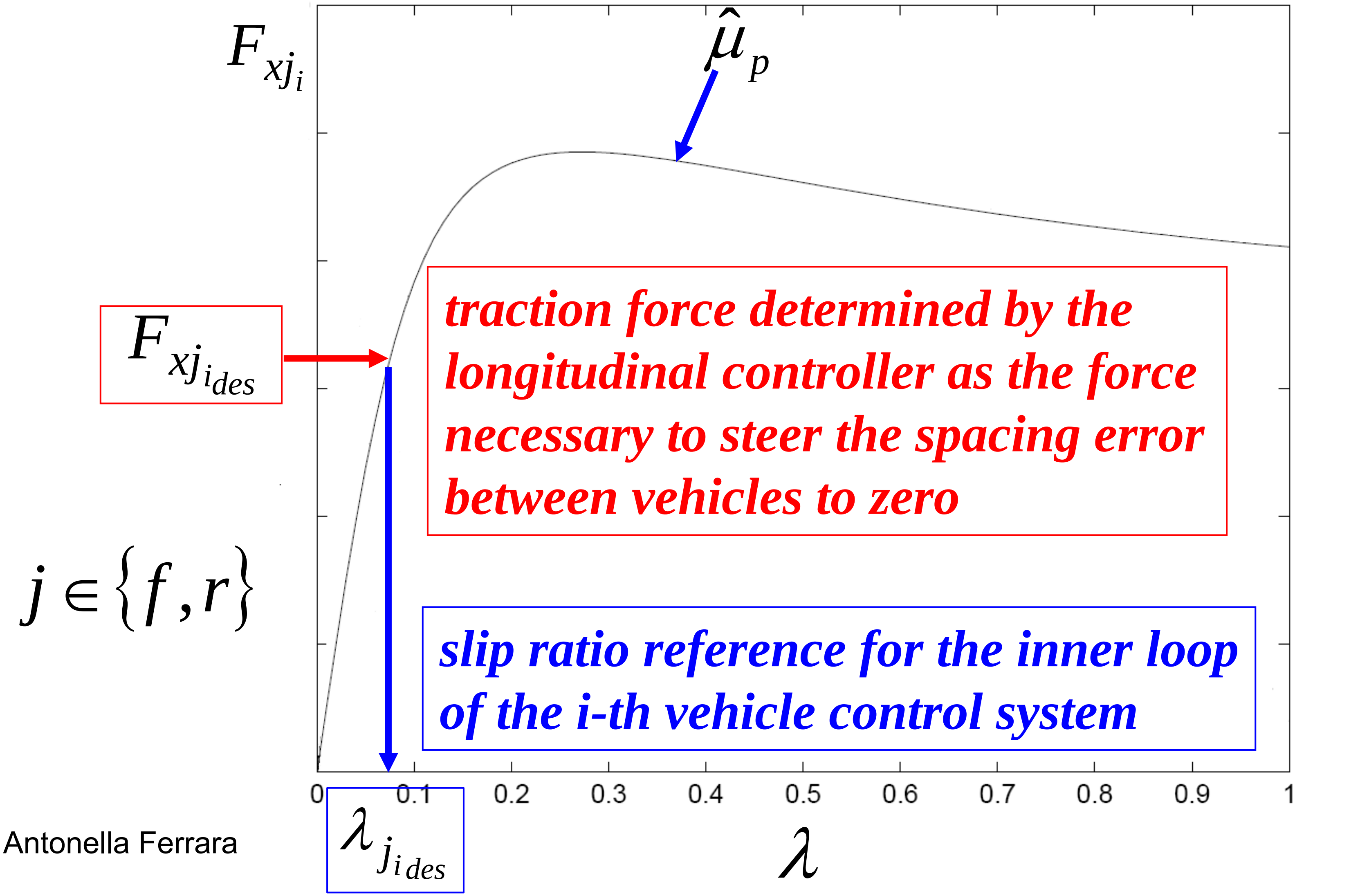
where

$$\dot{v}_{x_{i_{des}}} = \frac{1}{h}\int_{t_0}^{t} w_i(\tau)\,d\tau$$

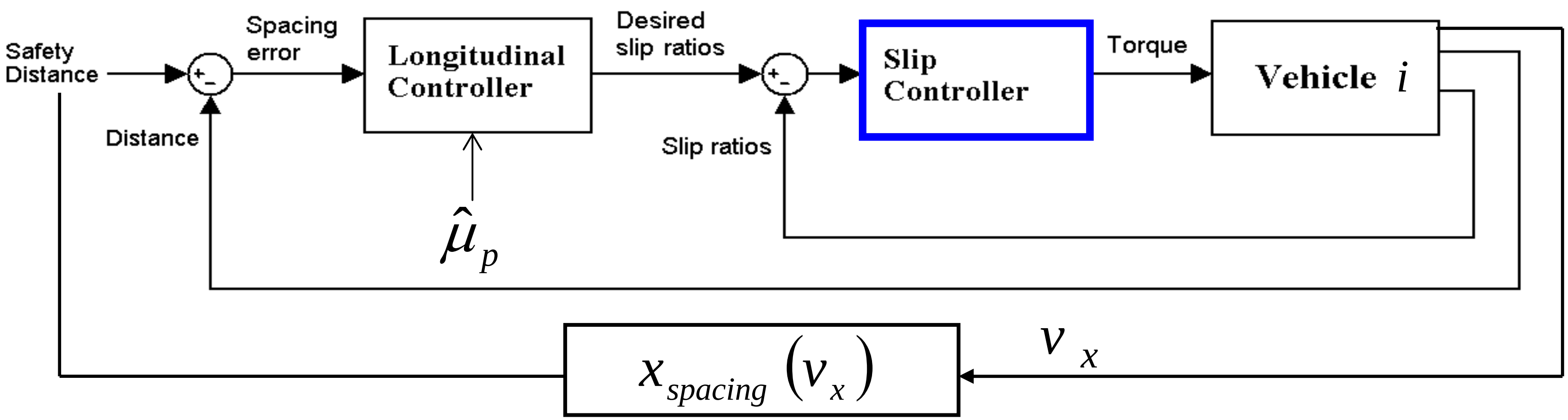
Note that, as usual in this context, it is assumed that the force distribution is described by

$$\frac{F_{xf_{i_{des}}}}{F_{xr_{i_{des}}}} = \frac{l_{r_i} + l_{h_i}\left(\dfrac{F_{loss\ i}}{m_i g} + \mu_i\right)}{l_{f_i} - l_{h_i}\left(\dfrac{F_{loss\ i}}{m_i g} + \mu_i\right)}$$

The slip reference for the inner loop of the *i*-th vehicle control system



The Slip Controller 1/2



Sliding variable: the **"slip error"** for each wheel *j* of the *i*-th vehicle

$$\boxed{s_j = \lambda_j - \lambda_{j_{des}} = \sigma_{j1}}$$

"auxiliary control"

Auxiliary system

$$\begin{cases} \dot{s}_j = \dot{\sigma}_{j_1} = \sigma_{j_2} \\ \ddot{s}_j = \dot{\sigma}_{j_2} = \overbrace{\dot{f}_j + \dot{h}_j(v_x)T_j - \ddot{\lambda}_{j_d}}^{\mathrm{F}_j(t)} + \overbrace{h_j(v_x)}^{\mathrm{g}_j(t)}\overbrace{\dot{T}_j}^{v_j(t)} \end{cases}$$

$$j \in \{f, r\}$$

(the subscript i is omitted !)

$$|\mathrm{F}_j(t)| < F_j \qquad 0 < G_{j_1} \leq \mathrm{g}_j(t) \leq G_{j_2}$$

The Slip Controller 2/2

It can be proved that, if the **auxiliary control signal** is chosen as

$$\boxed{v_j(t) = \dot{T}_j(t) = -\alpha_j \mathrm{N}_j \,\mathrm{sgn}\left(\sigma_{1_j}(t) - \frac{1}{2}\sigma_{1_{j\,MAX}}\right), \quad \sigma_{1_{j\,MAX}} = \sigma_{1_j}\Big|_{\sigma_{2_j}=0}}$$

with $\mathrm{N}_j > \max\left(\dfrac{F_j}{\alpha_j^* G_{j_1}}; \dfrac{4F_j}{3G_{j_1} - \alpha_j^* G_{j_2}}\right)$ and $j \in \{f, r\}$

then, the sliding variable and its first time derivative, $\sigma_{1_j} = s_j \quad \sigma_{2_j} = \dot{s}_j$, are steered to zero in a finite time.

As a consequence, the **slip error** relevant to each wheel of the ***i*-th vehicle** vanishes in a finite time

$$\boxed{s_j = \lambda_j - \lambda_{j_{des}} = 0}$$

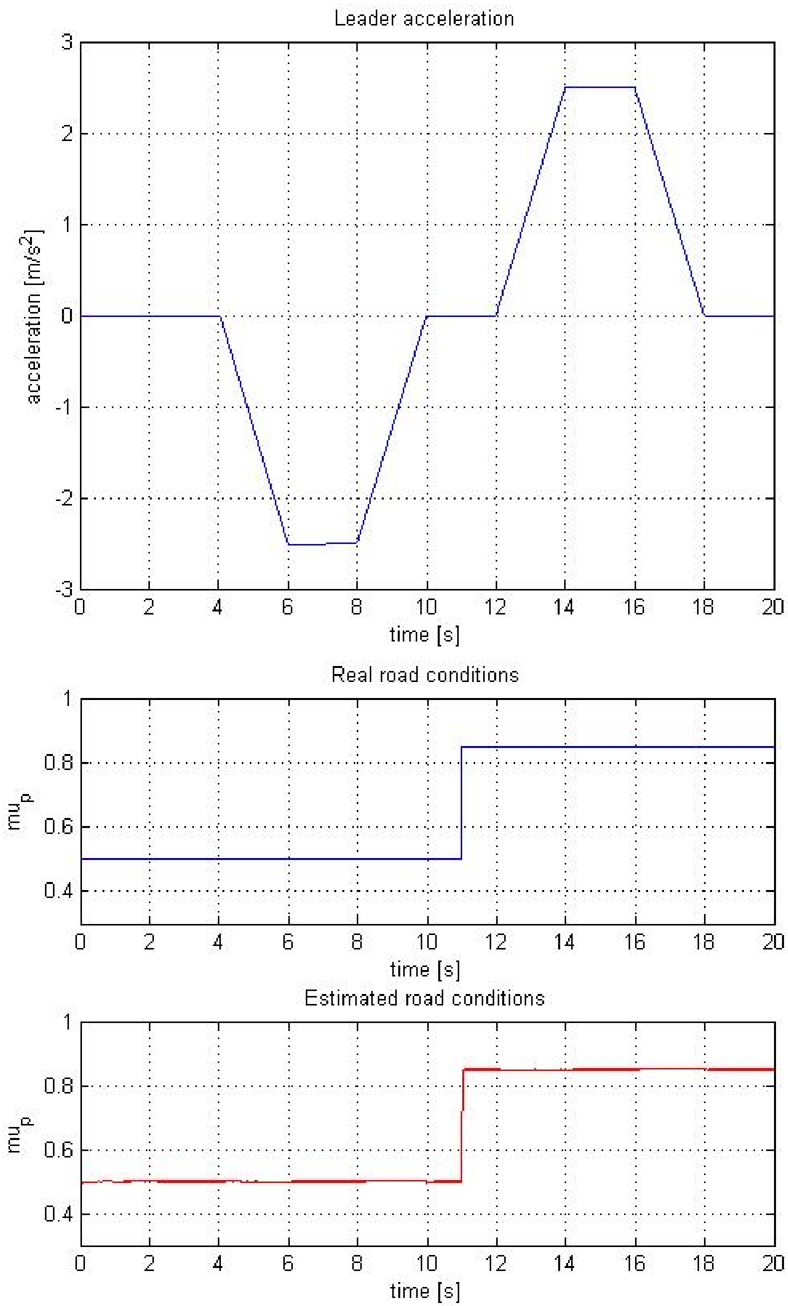
Simulation Test

Two vehicles, a leader and a follower.

TABLE I

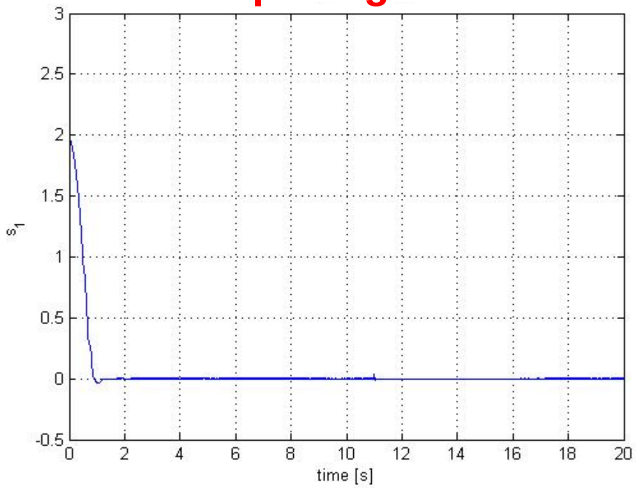
SIMULATION PARAMETERS

g	9.81	m/s^2
m	1202	kg
J_f	1.07	kgm^2
J_r	1.07	kgm^2
c_x	0.4	
f_{roll}	0.013	
r_f	0.32	m
r_r	0.32	m
l_f	1.15	m
l_r	1.45	m
l_h	0.65	m
$v_x(0)$	30	m/s
$\lambda_f(0)$	0.01	
$\lambda_r(0)$	0.01	

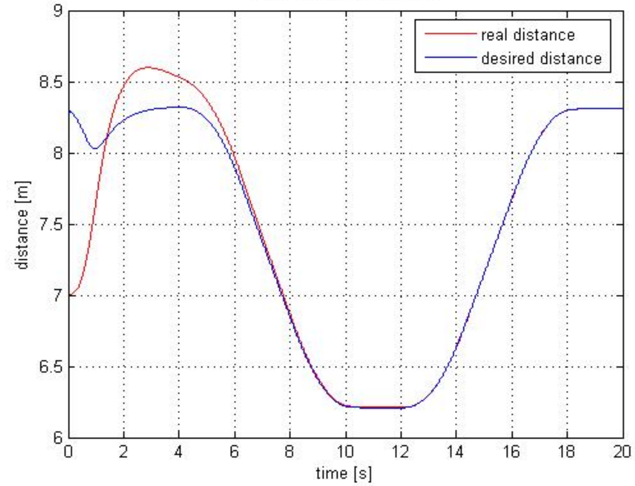


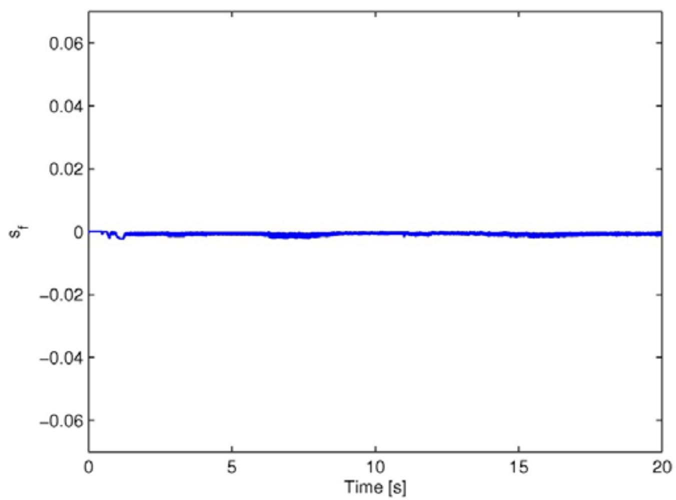
Simulation Results

Spacing error

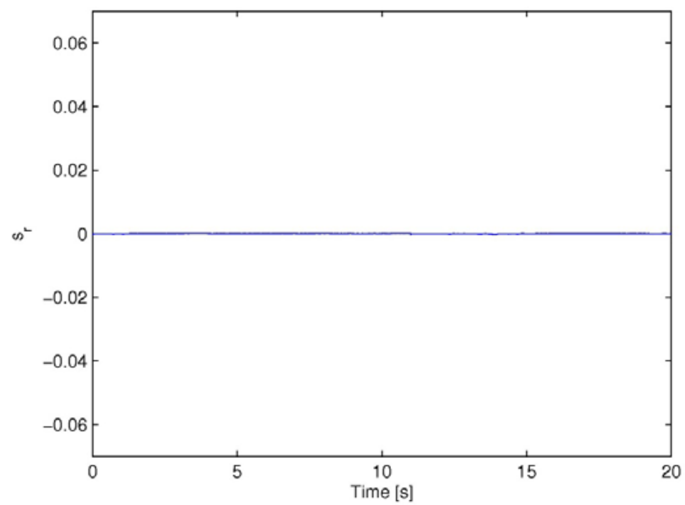


Inter-vehicular distance



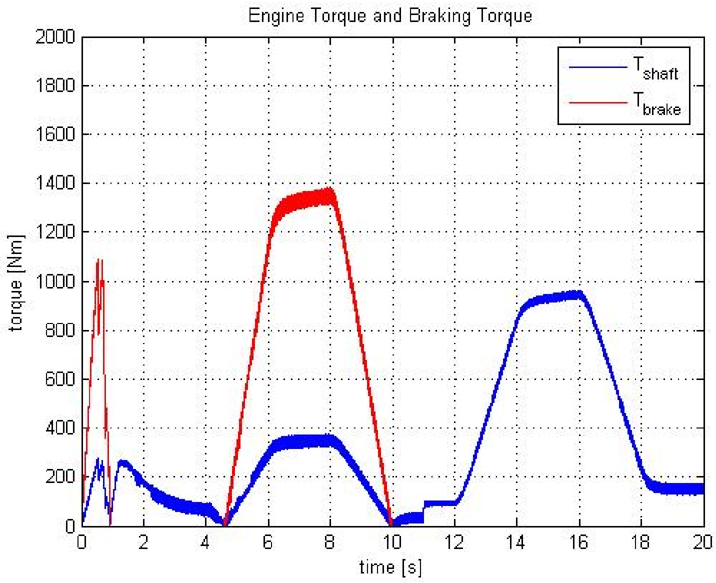


Slip error at the front wheel



Slip error at the rear wheel

Simulation Results

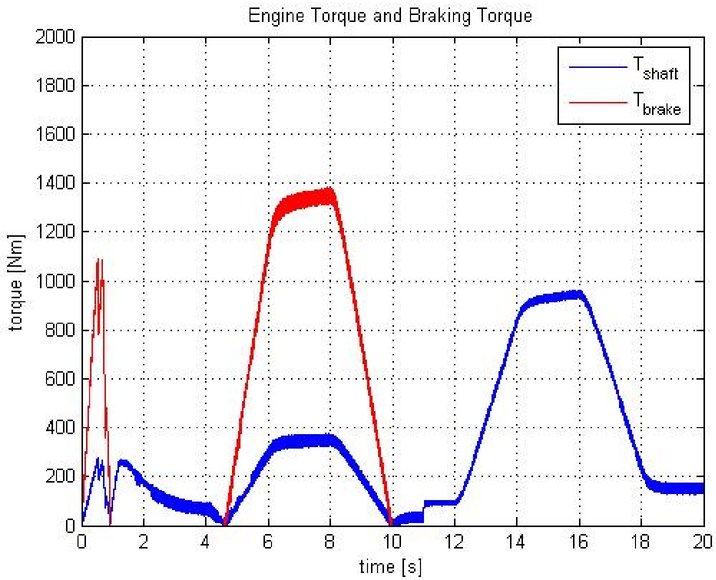


Braking torque and engine torque

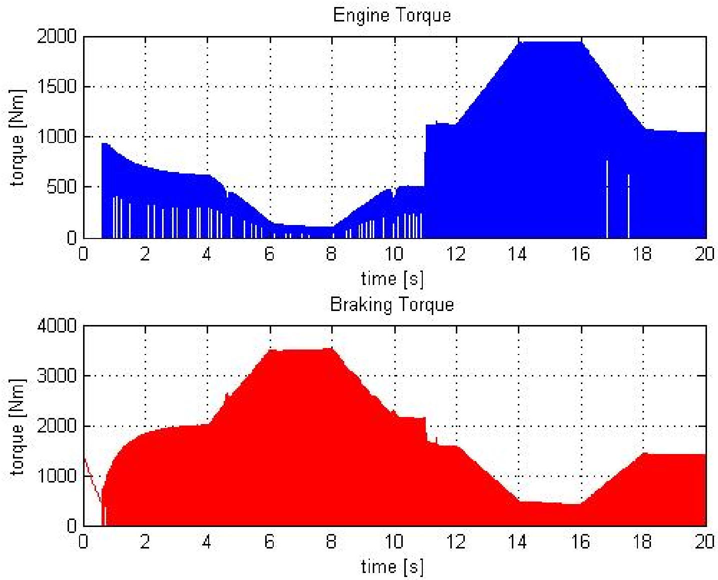
The total available braking torque and the engine torque exerted on the driving shaft can be determined from $T_j, \quad j \in \{f, r\}$
For instance, for front-wheel-driven (FWD) vehicles:

$$\boxed{\begin{aligned} T_f &= 0.5T_{shaft} - 0.3T_{brake} \\ T_r &= -0.2T_{brake} \end{aligned}}$$

Simulation Results

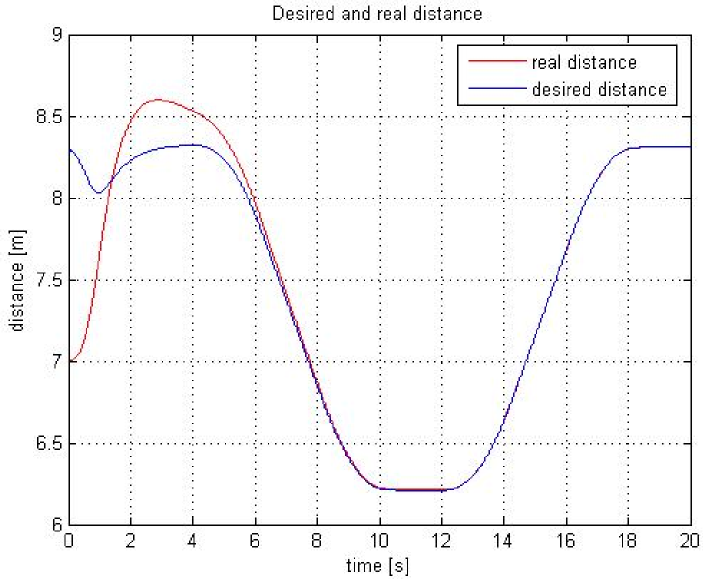


Braking torque and engine torque with the proposed ***second order sliding mode controller***

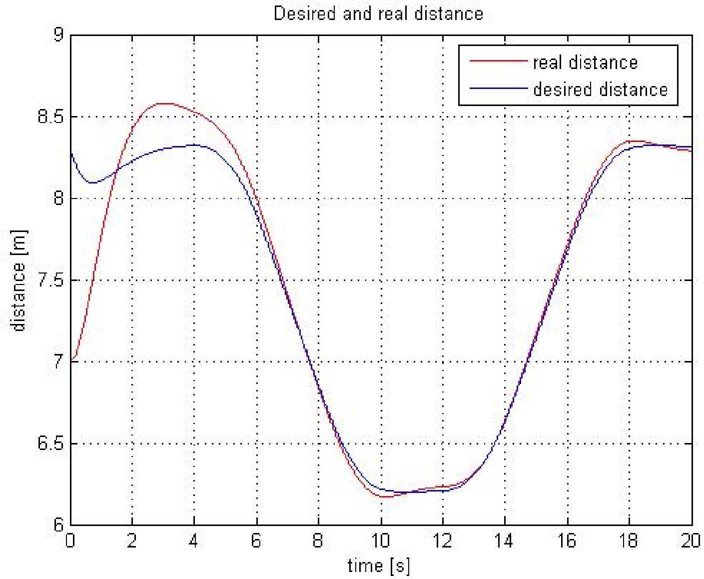


Braking torque and engine torque with a conventional first order sliding mode controller

Simulation Results



Inter-vehicular distance with the designed *second order sliding mode controller (ideal sliding mode!)*



Inter-vehicular distance with a conventional first order sliding mode controller with continuous approximation of the sign function (quasi sliding mode)

Braking Control of Two-Wheeled Vehicles via Switched Second Order Sliding Mode Control

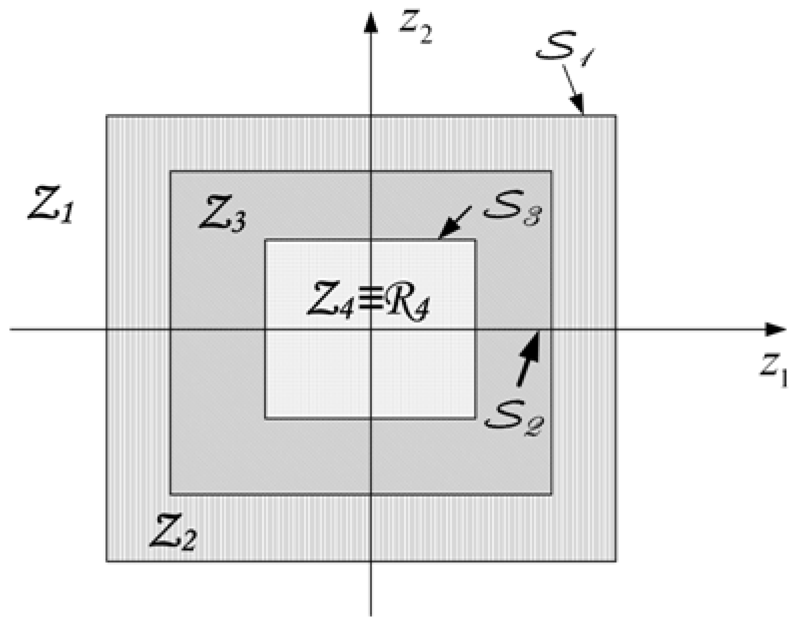
Braking Control

- **Braking control** is based on the regulation of the wheel slip (relative speed between wheel and center of mass). **In motorbikes the slip dynamics is dependent on the vehicle speed**.

- As an alternative to conventional adaptive control, we have proposed to apply the **switched formulation of second order sliding mode controllers (S-SOSM)**, since it can be useful in all contexts where different uncertainties and/or performance specs are associated with different **regions of the state space**.

MAIN IDEA:
- partition the state-space into different regions
- tune a dedicated S-SOSM controller for each of them

Switched SOSM: state-space partitioning



$$\mathcal{R}_i := \left\{ (z_1, z_2) : |z_1| \le \bar{z}_{1i} \text{ and } |z_2| \le \bar{z}_{2_i} \right\}$$

The state space $\mathcal{Z}$ is partitioned in ***k* regions** $\mathcal{R}_i$, *i=1,…, k*, all containing the **origin**, such that $\cup_i \mathcal{R}_i = \mathcal{Z}$ and with $\mathcal{R}_{i+1} \subset \mathcal{R}_i$.
Let us define as **switching surfaces** $\mathcal{S}_i = \partial \mathcal{R}_{i+1}$, *i=1,…,k-1.*

Switched SOSM: uncertainty description

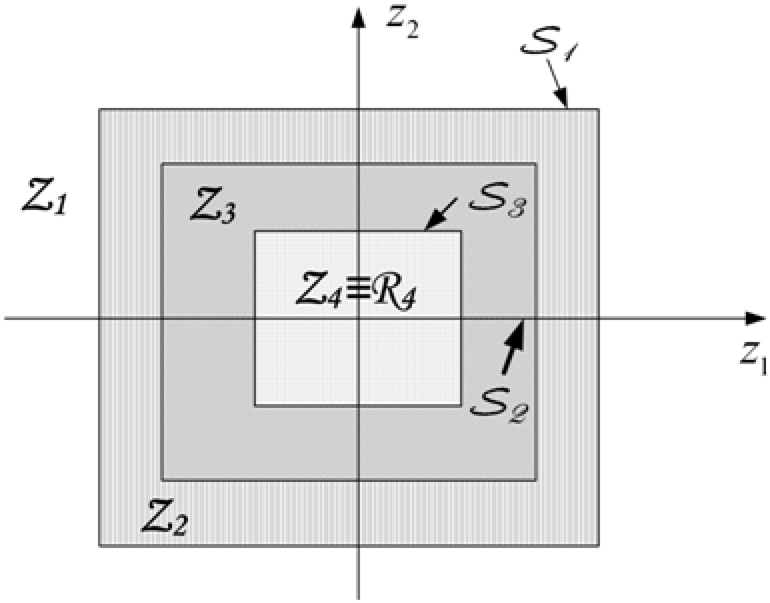
- Case 1: Outermost Region $\mathcal{Z}_1$

$$0 < G_{11} \leq g(z(t)) \leq G_{21}$$
$$|f(z(t)| < \mathcal{F}_1(z)$$

- Case 2: Inner Regions $\mathcal{Z}_i$, $i=2,\dots,k$

$$0 < \mathcal{G}_{1i}(z) \leq g(z(t)) \leq \mathcal{G}_{2i}(z)$$
$$|f(z(t)| < \mathcal{F}_i(z)$$

$$\mathcal{G}_{1i}(z) = G_{1i}(z) + \overline{G}_{1i}$$
$$\mathcal{G}_{2i}(z) = G_{2i}(z) + \overline{G}_{2i}$$
$$\mathcal{F}_i(z) = F_i(z)$$



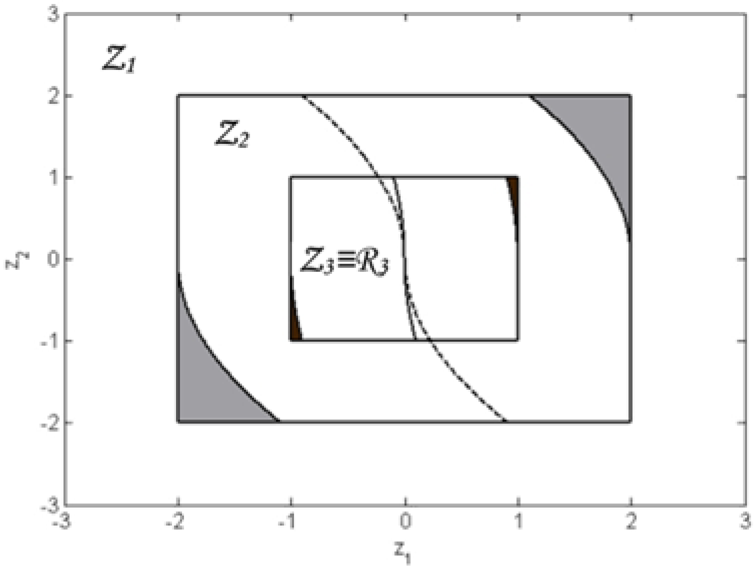
- As in the inner regions the state norm can be bounded, one can write:

$$0 < \overline{\mathcal{G}}_{1i} \leq g(z(t)) \leq \overline{\mathcal{G}}_{2i}$$
$$|f(z(t))| < \overline{\mathcal{F}}_i$$

Switched SOSM: control algoritms

- Two different algorithms:
  - **Full switched S-SOSM (FS-SOSM)** → both the gain V and α^* are adapted in the different regions (different uncertainties and different performance specifications)
  - **Gain switched S-SOSM (GS-SOSM)** → only the gain V is varied (uniform uncertainties in the whole state space but different performance specifications)

Switched SOSM: FS S-SOSM Control Algorithm



•**Outermost Region** $\mathcal{Z}_1$

1. Initialization: for $0 \leq t \leq t_{M1}$

$$v(t) = -\frac{1}{G_{11}}\left[\mathcal{F}_1(z(t)) + \nu\right]\text{sign}\,(z_1(0)),\ t = 0$$
$$v(t) = -\frac{1}{G_{11}}\left[\mathcal{F}_1(z(t)) + \nu\right]\text{sign}\,[z_1(t) - z_1(0)],\ 0 \leq t \leq t_{M_1}$$

2. For $t_{Mj} \leq t \leq t_{Mj+1}$ such that $z(t) \in \mathcal{Z}_1$

$$v(t) = -V_{M_j}\,\text{sign}\Big(z_1(t) - \beta_j z_{1_{M_j}}\Big)$$
$$V_{M_j} = \frac{\pi}{G_{11}}\left[\overline{\mathcal{F}}_1 + \frac{1}{3}\eta^2\right], \quad \pi > 1, \quad \beta_j = \max\left\{\frac{1}{2}, 1 - \frac{\eta^2}{2\left[\overline{\mathcal{F}}_1 + G_{21}V_{M_j}\right]}\right\}$$

Switched SOSM: FS S-SOSM Control (2)

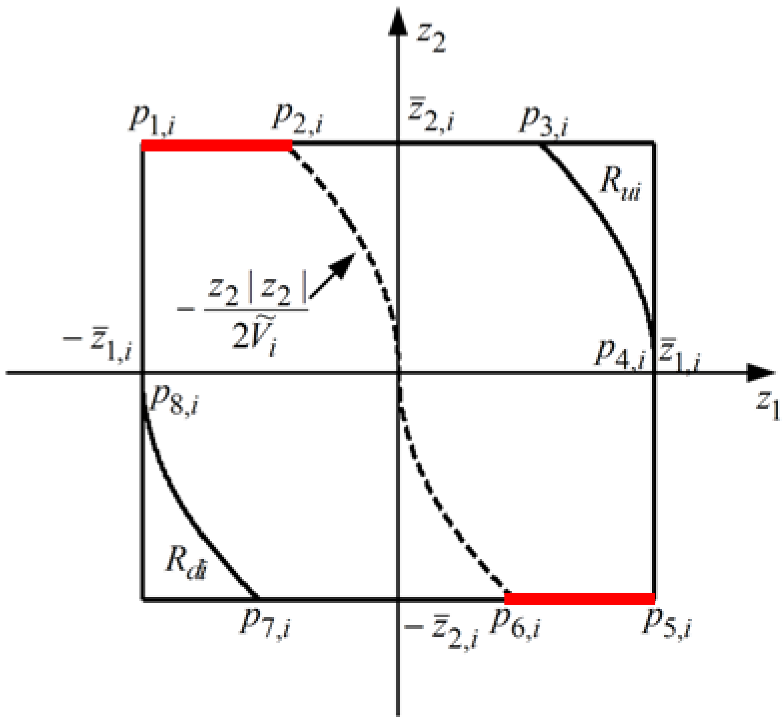
- **Inner Regions:**

If $z(t) \in \mathcal{Z}_i \setminus \{\overline{p_{1,i}p_{2,i}} \cup \overline{p_{5,i}p_{6,i}}\},\ i = 2, \ldots, k$

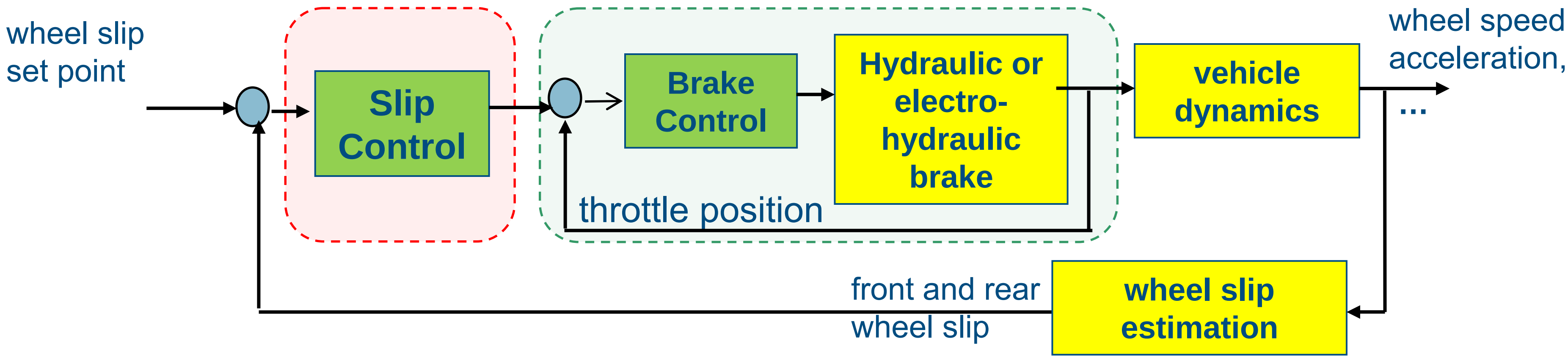
$$v(t) = -\alpha_i V_i \,\text{sign}\left(z_1 - \frac{1}{2} z_{Max}\right) \quad \alpha_i = \begin{cases} \alpha_i^* & \text{if} \quad [z_1 - \frac{1}{2} z_{Max}][z_{Max} - z_1] > 0 \\ 1 & \text{else,} \end{cases}$$

If $z(t) \in \{\overline{p_{1,i}p_{2,i}} \cup \overline{p_{5,i}p_{6,i}}\},\ i = 2, \ldots, k$

$$v_i(t) = -\alpha_i V_i \,\text{sign}\left(z_1(t) - \frac{1}{2} z_{Max}\right)$$
$$v_{i-1}(t) = \alpha_i V_i \,\text{sign}\left(z_1(t) - \frac{1}{2} z_{Max}\right),$$



Braking control of two wheeled vehicles



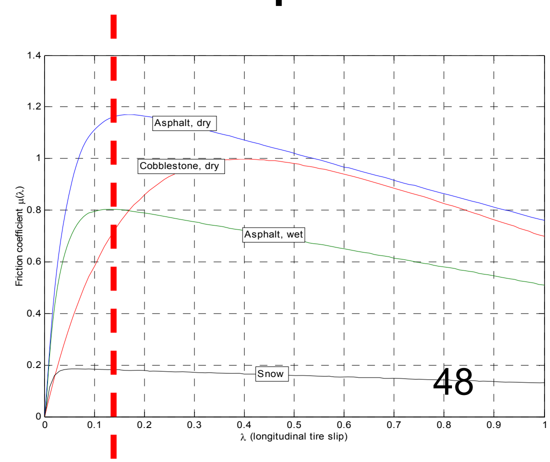
Main components:

- **<u>Servo-control loop for the actuator</u>: brake pressure control system**

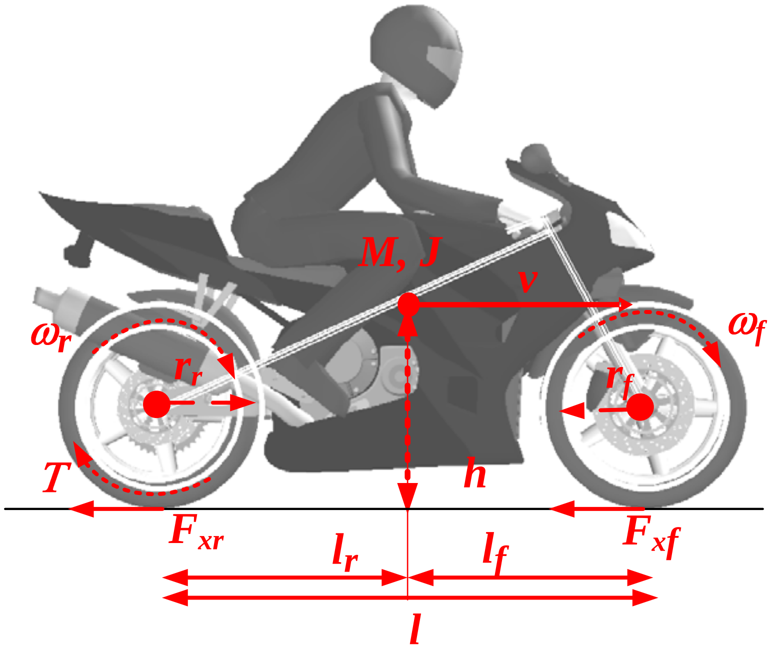
- **<u>Wheel slip estimation</u> (from indirect measurements): needed if front and rear brake is used. If front brake only (common on motorcycles), the rear wheel speed gives an estimate of the vehicle speed**

- **<u>Slip control</u>: the goal is to track a wheel slip target**

We have considered Braking Control on a straight line (target slip definition “easier”)



Braking control of two wheeled vehicles: vehicle model



$$\begin{cases} J\dot{\omega}_f = r_f F_{x_f} - T_{b_f} \\ J\dot{\omega}_r = r_r F_{x_r} - T_{b_r} \\ m\dot{v} = -F_{x_f} - F_{x_r} \end{cases}$$

$$F_{x_i} = F_{z_i}\mu(\lambda_i, \cancel{\alpha_{i_t}}; \vartheta),\ i = \{f, r\}$$

wheel slip

wheel sideslip (neglected as we consider straight line maneuvers)

tire/road model parametrization

- The wheel slip in braking is defined as

$$\lambda_i = \frac{v - \omega_i r}{\omega_i r}$$

$$F_{z_f} = \frac{mgl_r}{l} - \frac{mh}{l}\dot{v} = W_f - \Delta_{F_z}\dot{v}$$

$$F_{z_r} = \frac{mgl_f}{l} + \frac{mh}{l}\dot{v} = W_r + \Delta_{F_z}\dot{v}$$

Load transfer between front and rear wheels –> proportional to longitudinal acceleration

Tire/Road Interaction

Year	Model Name	Model Properties	Features
	Piecewise Linear Model	Empirical	1. Cannot accurately fit curves 2. Easy to identify
1993	Burckhardt Model	Semi-Empirical	1. Can accurately fit curves 2. Has some revised formula
1994	Rill Model	Semi-Empirical	Easy to identify
1987	Magic Formula	Semi-Empirical	1. Can accurately fit curves 2. Has lot of revised formula 3. Can employ different factors
1977	Dahl Model	Analytical	1. Can describe Coulomb friction. 2. Can produces smooth transition around zero velocity
1991	Bliman-Sorine Model	Analytical	Can capture the Stribeck effect in addition to Dahl model
1995	LuGre Model	Analytical	Can combine pre-sliding & sliding in addition to Bliman-Sorine Model

Table from: ***Li Li, Fei-Yue Wang, and Qunzhi Zhou, "Integrated Longitudinal and Lateral Tire/Road Friction Modeling and Monitoring for Vehicle Motion Control", IEEE Trans. On Intelligent Transportation Systems, Vol. 7, No. 1, March 2006.***

The Tire/Road Interaction

The **“Burkhardt model”** describes the tire-road friction as a function of the wheel slip

$$\boxed{\mu(\lambda;\vartheta) = \vartheta_1(1 - e^{-\lambda\vartheta_2}) - \lambda\vartheta_3}$$

By changing the values of the parameters ϑ_r different road conditions can be modeled

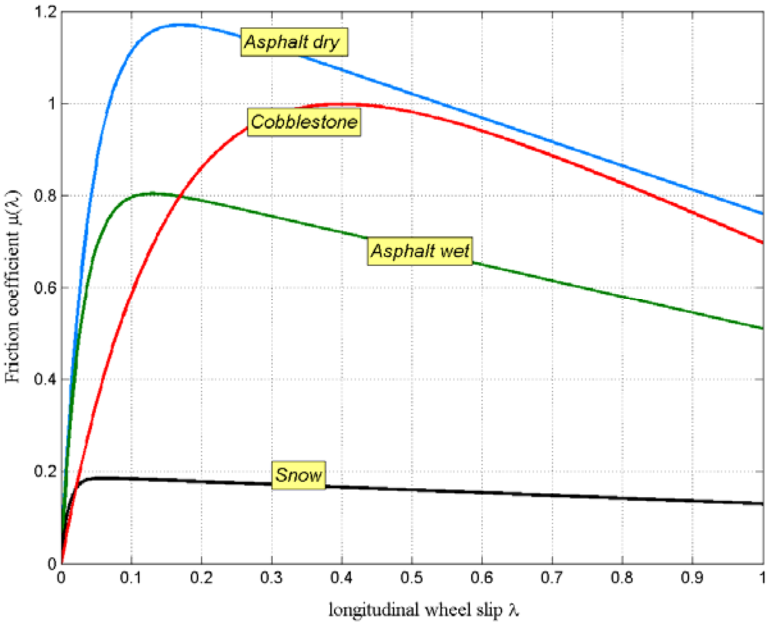
Note: the friction curve model, together with the fact that

$$F_{x_i} = F_{z_i}\,\mu(\lambda_i;\vartheta)$$

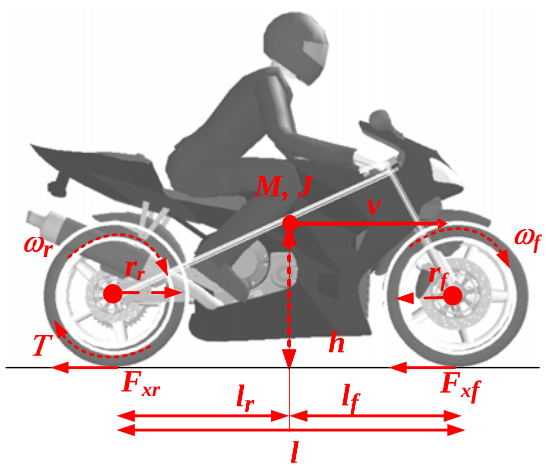
implies that longitudinal forces are bounded $|F_{xi}| \leq \Psi,\ i \in \{f, r\}.$

Due to tire relaxation dynamics, also the forces first time derivatives are bounded

$$|\dot{F}_{xi}| \leq \Gamma,\ i \in \{f, r\}$$



Braking Dynamics: slip dynamics



Wheel speed and wheel slip are linked by an algebraic relationship → can use the wheel slip as state variable for wheel dynamics

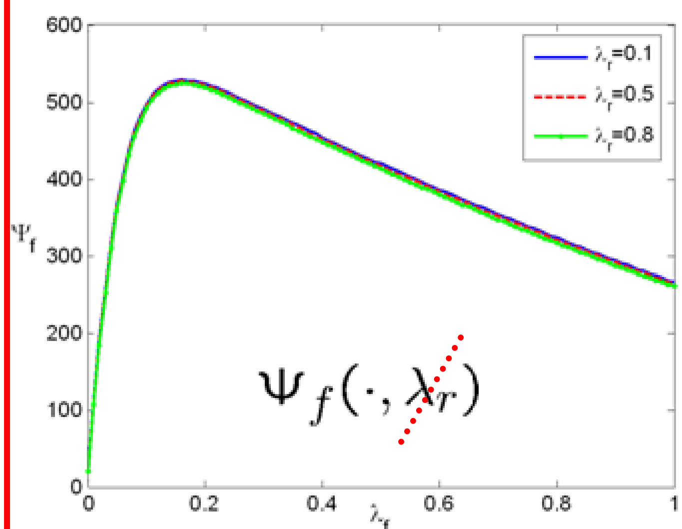
$$\begin{cases} \dot{\lambda}_f = -\frac{r}{Jv}\left(\Psi_f(\lambda_f,\lambda_r) - T_{b_f}\right) \\ \dot{\lambda}_r = -\frac{r}{Jv}\left(\Psi_r(\lambda_f,\lambda_r) - T_{b_r}\right) \\ \dot{v} = -\frac{W_f\mu(\lambda_f)+W_r\mu(\lambda_r)}{m-\Delta_{F_z}(\mu(\lambda_f)-\mu(\lambda_r))} \end{cases}$$

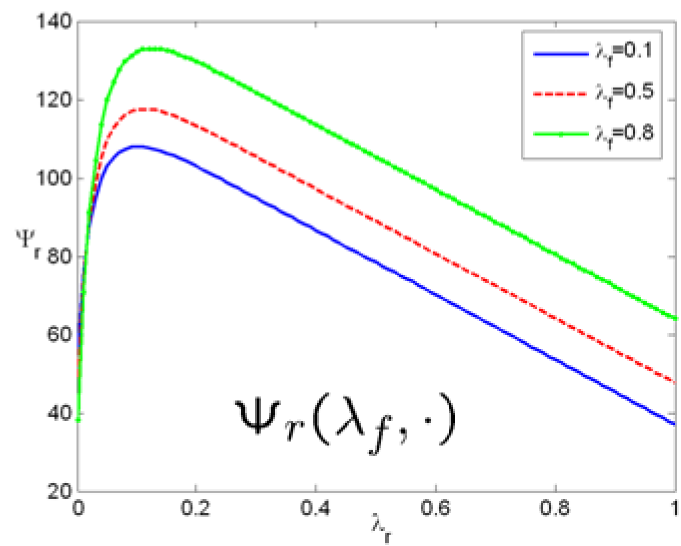
The dependence of Ψ_r on λ_f can be easily managed within a SM framework → Ψ_r bounded for all λ_f and this is all one needs to know for designing a SM controller

Two SISO controllers can be designed, and the coupling between front and rear wheel only affects the bounds on the uncertainties

$$\Psi_f(\lambda_f,\lambda_r) = \left[r(W_f - \Delta_{F_z}\dot{v})\mu(\lambda_f) - \frac{J}{r}(1-\lambda_f)\dot{v}\right]$$

$$\Psi_r(\lambda_f,\lambda_r) = \left[r(W_r + \Delta_{F_z}\dot{v})\mu(\lambda_r) - \frac{J}{r}(1-\lambda_r)\dot{v}\right]$$





S-SOSM Wheel Slip Controller

The selected **sliding variable** are the **slip tracking errors**

$$\boxed{s_i = \lambda_i - \lambda_i^*,\ i \in \{r, f\}}$$

Auxiliary system

$$\begin{cases} \dot{s}_i &= \dot{\lambda}_i - \dot{\lambda}_i^* \\ \ddot{s}_i &= \varphi_i + h_i \dot{T}_i, \end{cases}$$

"auxiliary control signal": it is the control law *v(t)* of the S-SOSM algorithm

$$\varphi_i := \frac{r\omega_i}{v^2}\ddot{v} + 2\frac{r\dot{\omega}_i\dot{v}}{v^2} - 2\frac{r\omega_i\dot{v}^2}{v^3} - \frac{r^2}{Jv}\dot{F}_{xi} - \ddot{\lambda}_i^*$$

$$h_i := \frac{r}{Jv}$$

Both φ_i and h_i are fns of v!

S-SOSM Wheel Slip Controller (2)

Auxiliary system

$$\begin{cases} \dot{s}_i = \dot{\lambda}_i - \dot{\lambda}_i^* \\ \ddot{s}_i = \varphi_i + h_i \dot{T}_i, \end{cases}$$

"auxiliary control signal"

$$\boxed{|\varphi_i| \leq \Phi_i(v, \omega_i, T_{bi})}$$

$$0 < \Gamma_{i1}(v, \omega_i) \leq h_i \leq \Gamma_{i2}(v, \omega_i)$$

Only the knowledge of the **upper-bounds** is necessary to design the control law

Simulation Results

Results on a Simulink model including brakes dynamics, tire elasticity and tire relaxation (definitely more complex than that used for the design!)

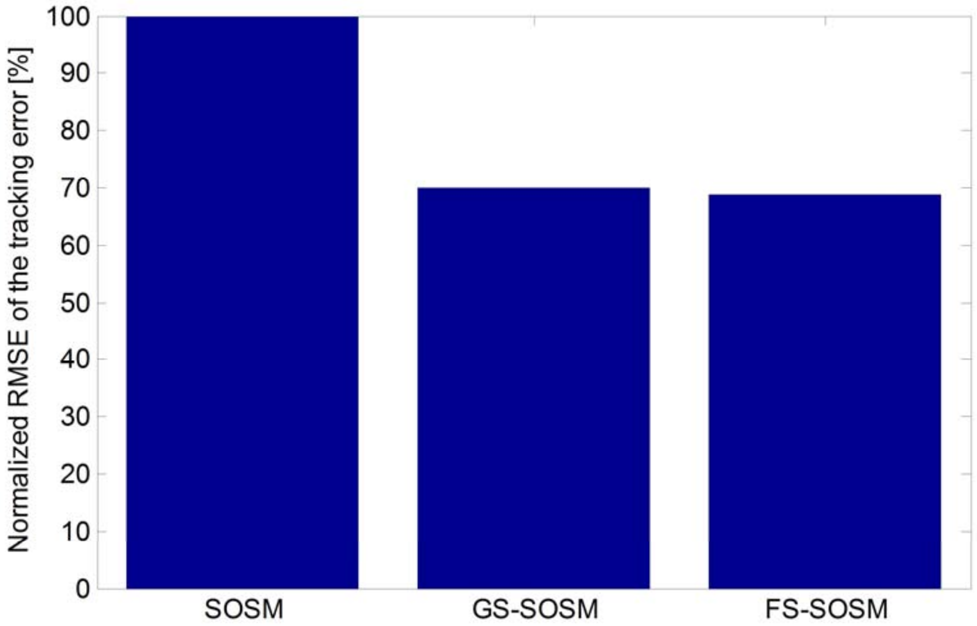
Braking maneuver with set-point step-changes of width 0.05 from $\lambda^*=0$ to $\lambda^*=0.2$

Three algorithms compared: Suboptimal SOSM, FS-SOSM, GS-SOSM

Simulation Results (2)

$$J_s = \sqrt{\sum_{i=1}^{N} s(i)^2 / N}$$

$$J_{s_{Norm}} = 100 \frac{J_s}{\max_{i=1,\dots,3}\{J_{s_i}\}}$$



The switched algorithms allow for a 30% improvement with respect to the standard SOSM one

The small difference between FS-SOSM and GS-SOSM probably due to the fact that Js captures performance and both algorithms behave similarly in this respect

Torque Vectoring Control in Fully Electric Vehicles via SMC

Torque Vectoring via SMC

• The **torque-vectoring control of a four-wheel-drive fully electric vehicle with in-wheel drivetrains** has been studied in recent years, and at **ACC14** an ISM control based solution has be presented in collaboration with: UNIVERSITY OF SURREY

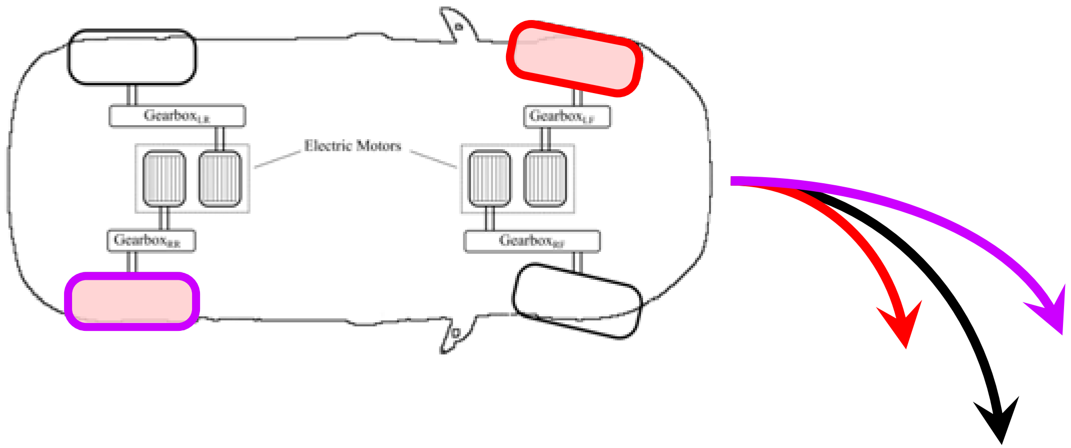
• The **ISM controller** is easy to tune and robust with respect to a significant set of uncertainties and disturbances.

• Its low complexity and other positive features make it appropriate for **practical implementations**.

European Project E-VECTOORC

Vehicle Concept and Layout	Powertrain Design and Safety	Brake Design and EM-compatibility	Vehicle Dynamics and Control
JAGUAR, LAND ROVER, SKODA AUTO, VW	inverto, FLANDERS' DRIVE	TRW, ita, cidaut Transport and Energy Research and Development Foundation	UNIVERSITY OF SURREY, ILMENAU UNIVERSITY OF TECHNOLOGY, virtual vehicle

Electronic Stability Program (ESP) VS Torque-Vectoring (TV)



ESP

- Only when $|r_{ref} - r| > th.$
- ON-OFF control
- By means of:
  - Friction brakes
  - Wheel torques (-)

TV

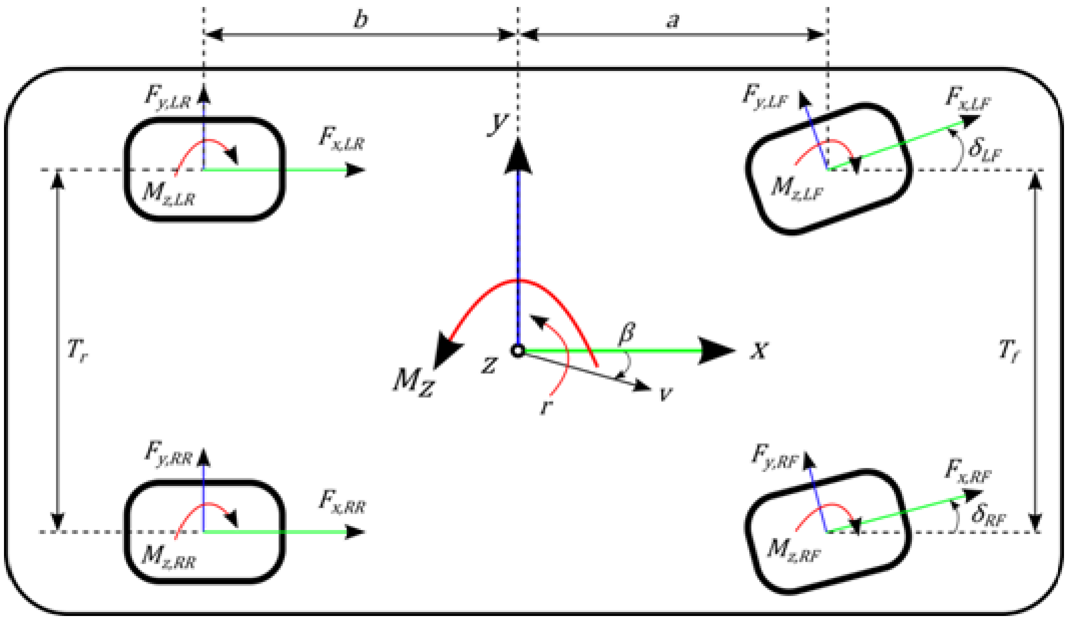
- In any condition (v, a_x, a_y, δ)
- At any instant of time
- By means of:
  - Friction brakes
  - Wheel torques (+/-)

Why Sliding Mode for Yaw Rate Control?

- **Conventional yaw rate control systems: PID + FF**
  - Good tracking performance and large bandwidth of the closed-loop system
  - Non robust and smooth enough in critical conditions (high lateral acceleration)
- **Many other approaches have been investigated (some examples):**
  - Internal Model Control;
  - MPC/Linear Quadratic Control;
  - Optimal Control based on LMI.
- **Major motivations for using *Integral Sliding Mode* Control:**
  - Ease of implementation (few parameters only);
  - High level of robustness against unmodeled dynamics and external disturbances, even when implemented with sampling times typical of real automotive control applications (experimental tests support this claim).

The Vehicle Model

The proposed controller has been designed considering the vehicle yaw dynamics



$$\dot{x} = f(x) + B(x)u + h(x,t) \longrightarrow \dot{r} J_z = \left(-F_{y,RF}\sin\delta_{RF} + F_{y,LF}\sin\delta_{LF} + F_{x,RF}\cos\delta_{RF} - F_{x,LF}\cos\delta_{LF}\right)\cdot\frac{T_f}{2} +$$

$$\left(F_{y,RF}\cos\delta_{RF} + F_{y,LF}\cos\delta_{LF} + F_{x,RF}\sin\delta_{RF} + F_{x,LF}\sin\delta_{LF}\right)\cdot a +$$

$$\left(F_{y,RR} + F_{y,LR}\right)\cdot b + \left(F_{x,RR} - F_{x,LR}\right)\cdot\frac{T_r}{2} -$$

$$M_{z,RF} - M_{z,LF} - M_{z,RR} - M_{z,LR}$$

Design of the proposed ISM controller

In the previous model, the overall yaw moment is the control variable

Yaw Rate Error Model

$$-\dot{r}_{err} = \dot{r} - \dot{r}_{ref} = k(r,\beta,\delta) - \dot{r}_{ref} + \frac{1}{J_z} M_{z,ISM} + \frac{1}{J_z} M_{z,dist}$$

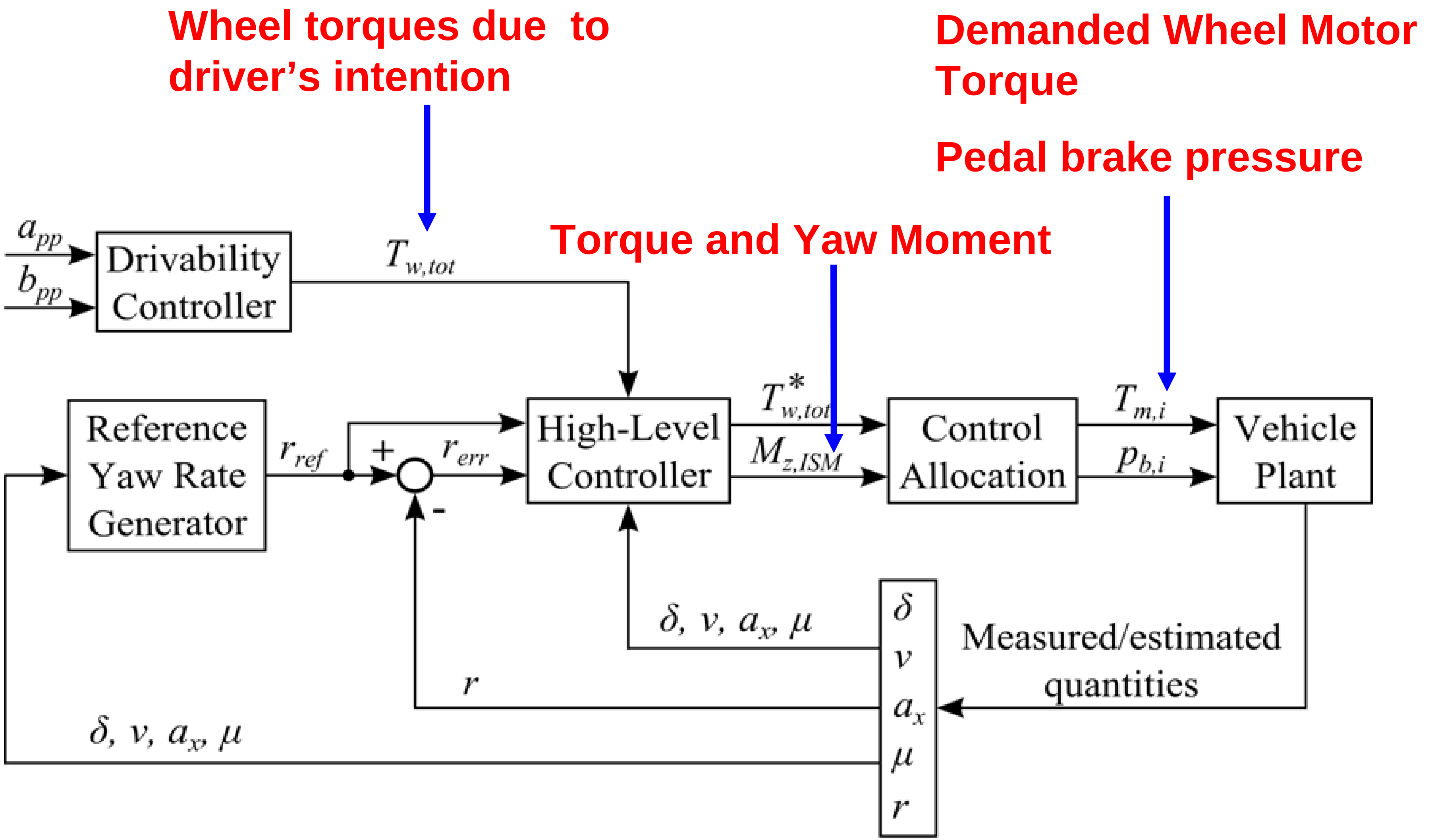
Unknown part: $h = k(r,\beta,\delta) + \frac{1}{J_z} M_{z,dist}$

Known part: $-\dot{r}_{ref}$

Yaw Acceleration contribution due to lateral forces and self-aligning moments

$$k(r,\beta,\delta) = \frac{1}{J_z}\left(-F_{y,RF}\sin\delta_{RF} + F_{y,LF}\sin\delta_{LF}\right)\cdot\frac{T_f}{2} + \left(F_{y,RF}\cos\delta_{RF} + F_{y,LF}\cos\delta_{LF}\right)\cdot a - \left(F_{y,RR} + F_{y,LR}\right)\cdot b - M_{z,RF} - M_{z,LF} - M_{z,RR} - M_{z,LR}$$

The Yaw Rate Control Scheme



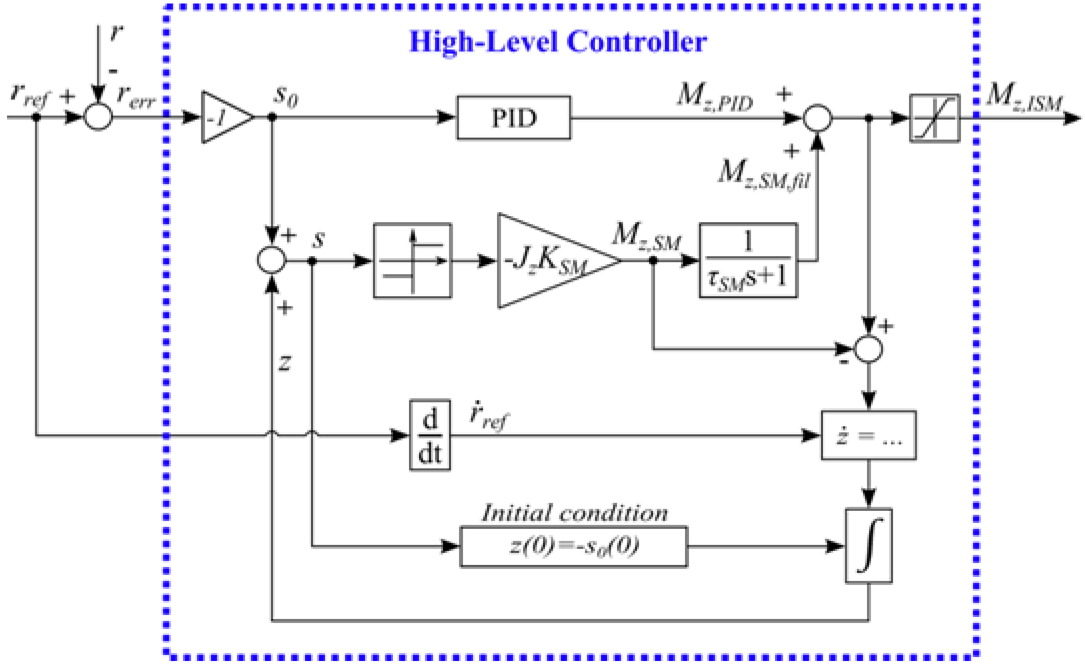
Integral Sliding Mode Control

$$M_{z,ISM} = M_{z,PID} + M_{z,SM,fil}$$

$$M_{z,SM} = -J_z K_{SM} \, sign(s)$$

$$\tau_{SM} \dot{M}_{z,SM,fil} + M_{z,SM,fil} = M_{z,SM}$$

$$s = s_0 + z$$



Where $\dot{z} = -\dfrac{\partial s_0}{\partial(-r_{err})}\left[-\dot{r}_{ref} + \dfrac{1}{J_z}\left(M_{z,ISM} - M_{z,SM}\right)\right]$ with the initial condition $z(0) = -s_0\left(r_{err}(0)\right)$

The sliding mode starts from the first sampling instant, without any reaching-phase transient

Controller Parameters Design

Only two parameters needs to be tuned: K_{SM} and τ_{SM}

- ***η*-reaching condition** $s\dot{s} < -\eta|s|$ fulfilled for any value $K_{SM} > J_z \cdot |h_{\max} + \eta|$
- a sequence of step steer maneuvers at *v=90 km/h* (high critical situation) has provided the upper bound $J_z \cdot |h_{\max}| \cong 9{,}000\,Nm$
- a conservative value has been chosen: $K_{SM} = 15{,}000\,Nm$
- The value $\tau_{SM} = 0.05s$ has been selected in order to avoid the **chattering effect**, so that the yaw moment generated by the high-level controller does not induce vibrations perceivable by the passengers.

The Show Case

The Show Case

- The performance assessment has been carried out using an accurate IPG CarMaker model of the vehicle (validated on the basis of experimental tests at the **Lommel Proving Ground in Belgium**)

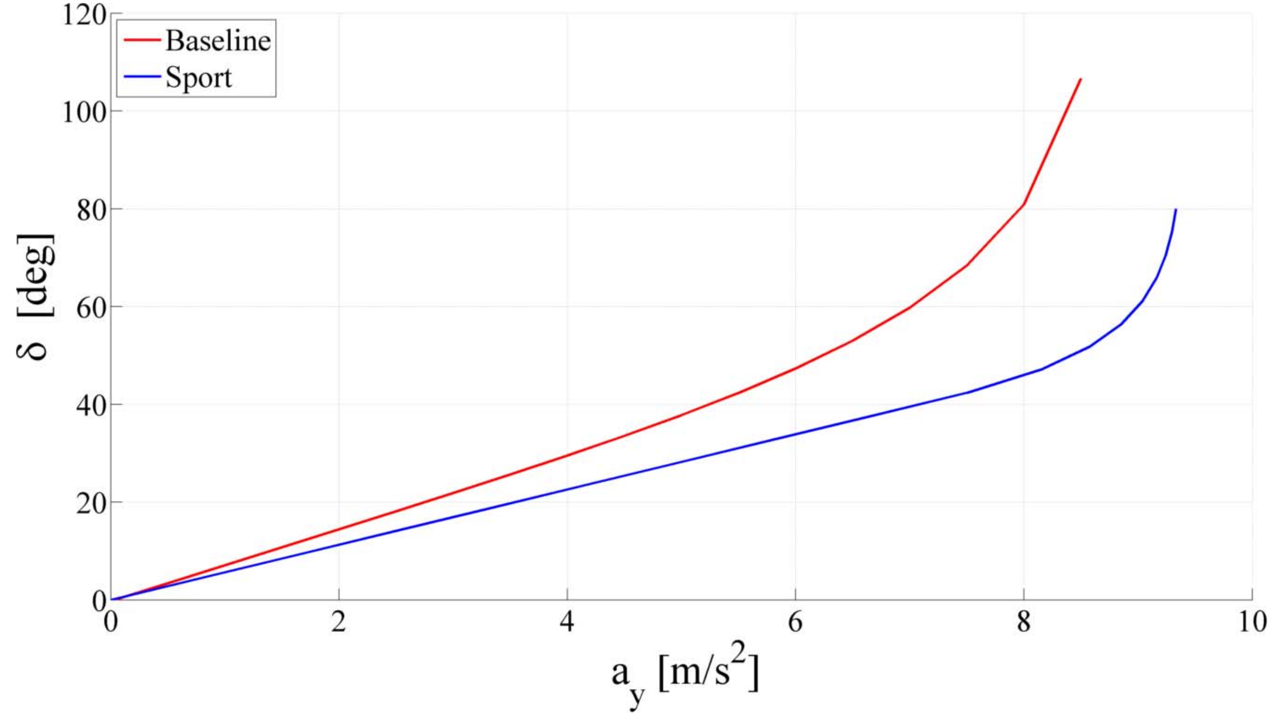
- Two conventional test maneuvers have been considered:

  - **Ramp Steer Meneuver**

  - **Step Steer Maneuver**

- The controlled vehicle (**“Sport Mode”**) has been compared with the passive vehicle (**“Baseline Mode”**)

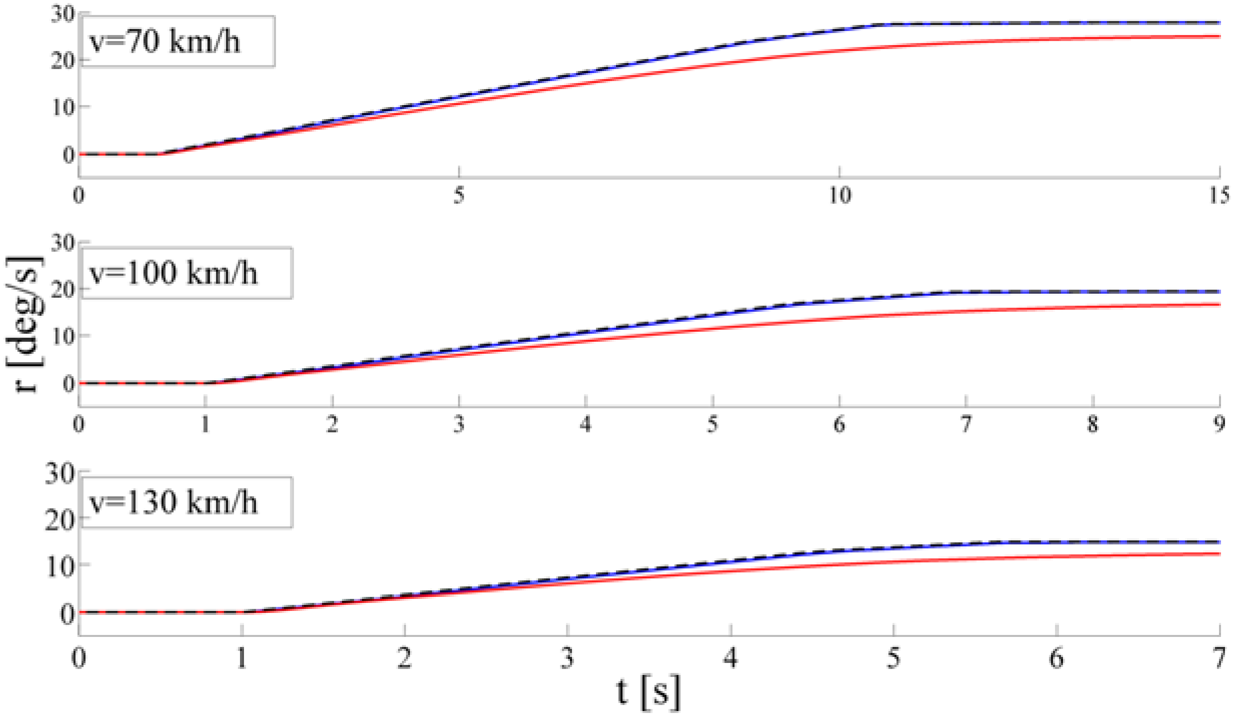
Ramp Steer Maneuver (I)



Benefits of the adoption of the ISM yaw rate controller:

- **lower value of the understeer gradient;**
- **extension of the linear region of the vehicle understeer characteristic;**
- **higher values of lateral acceleration.**

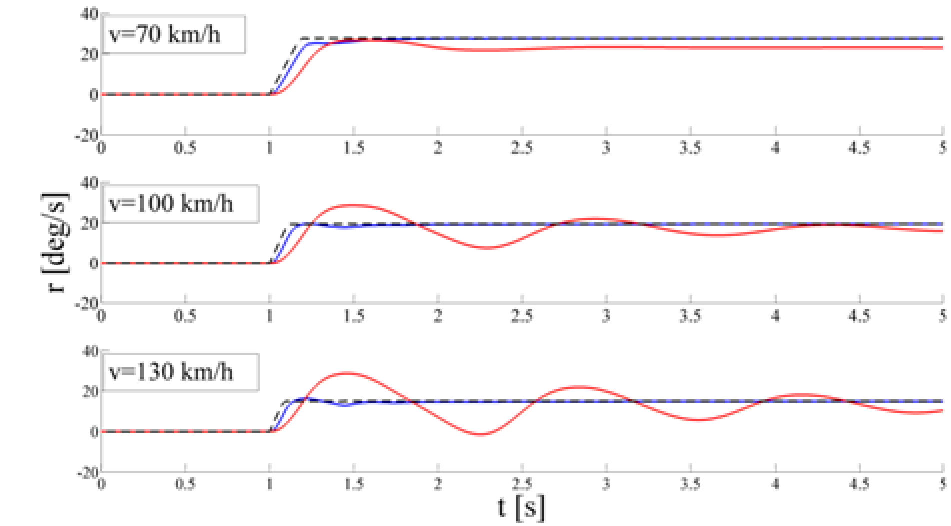
Ramp Steer Maneuver (II)

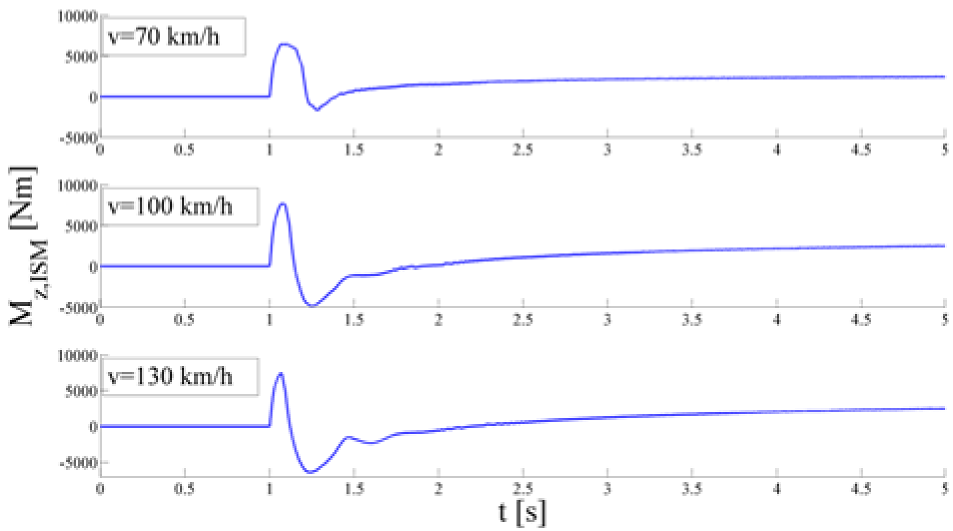


Reference yaw rate tracking performance at different speeds.

The reference (dashed dark line) and the yaw rate of the controlled vehicle (blue line) are practically overlapped.
The red line is the yaw rate of the passive vehicle.

Step Steer Maneuver

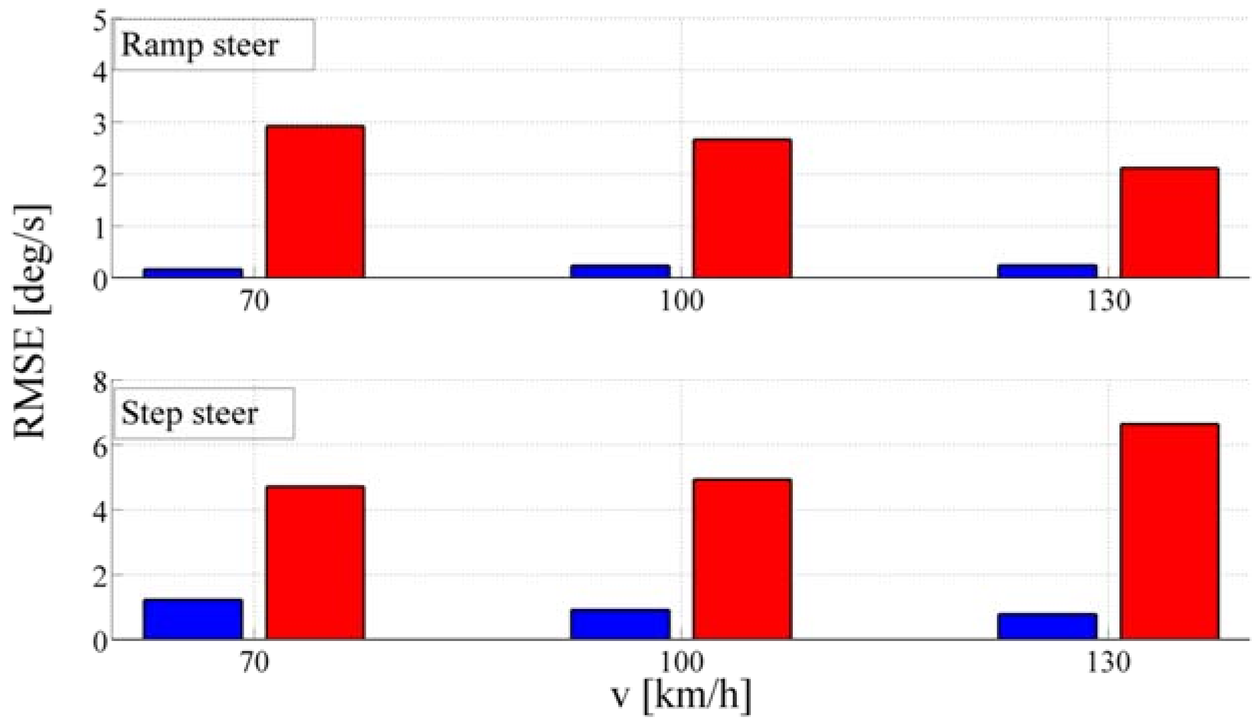




- **Evident degradation of the passive vehicle tracking performance at increasing speeds**
- **Good tracking of the yaw rate reference exhibited by the controlled vehicle, with very short rise-time and negligible overshoot.**

- **Obtained by means of a control signal perfectly acceptable in automotive applications.**

Summary Results



BLUE: Controlled veh. RED: Passive veh.

$$RMSE = \sqrt{\frac{1}{t_{end} - t_{in}} \int_{t_{in}}^{t_{end}} (r_{err})^2 \, dt}$$

Some Final Remarks

- **Sliding mode control** is an interesting methodology for its simplicity and robustness features, but it may be non appropriate to be applied, in its conventional formulation, to the automotive context due to the discontinuous control variable.

- The **second order sliding mode controller** generates continuous control actions, thus limiting the vibrations it can induce and propagate throughout the vehicle subsystems because of chattering effects.

- The **switched version** of Suboptimal SOSM control law allows to tune the controller parameters taking into account the vehicle speed. For this reason it is particularly appropriate for motorbikes, the slip dynamics of which strictly depends on speed.

- **Integral SMC** can also be an effective solution.

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Last Part of the Lecture

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